Calculating Fermion masses in
Superstring Derived Standard–like Models

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ABSTRACT

One of the intriguing achievements of the superstring derived standard–like models in the free fermionic formulation is the possible explanation of the top quark mass hierarchy and the successful prediction of the top quark mass. An important property of the superstring derived standard–like models, which enhances their predictive power, is the existence of three and only three generations in the massless spectrum. Up to some motivated assumptions with regard to the light Higgs spectrum, it is then possible to calculate the fermion masses in terms of string tree level amplitudes and some VEVs that parameterize the string vacuum. I discuss the calculation of the heavy generation masses in the superstring derived standard–like models. The top quark Yukawa coupling is obtained from a cubic level mass term while the bottom quark and tau lepton mass terms are obtained from nonrenormalizable terms. The calculation of the heavy fermion Yukawa couplings is outlined in detail in a specific toy model. The dependence of the effective bottom quark and tau lepton Yukawa couplings on the flat directions at the string scale is examined. The gauge and Yukawa couplings are extrapolated from the string unification scale to low energies. Agreement with $\alpha_{\rm strong}$, $\sin^2\theta_W$ and $\alpha_{\rm em}$ at M_Z is imposed, which necessitates the existence of intermediate matter thresholds. The needed intermediate matter thresholds exist in the specific toy model. The effect of the intermediate matter thresholds on the extrapolated Yukawa couplings in studied. It is observed that the intermediate matter thresholds also help to maintain the correct b/τ mass relation. It is found that for a large portion of the parameter space, the LEP precision data for $\alpha_{\rm strong}$, $\sin^2\theta_W$ and $\alpha_{\rm em}$, as well as the top quark mass and the b/τ mass relation can all simultaneously be consistent with the superstring derived standard–like models. Possible corrections due to the supersymmetric mass spectrum are studied as well as the minimization of the supersymmetric Higgs potential. It is demonstrated that the calculated values of the Higgs VEV ratio, $\tan\beta = v_1/v_2$, can be compatible with the minimization of the one–loop effective Higgs potential.

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1. Introduction

One of the most important problems in elementary particle physics is the origin of fermion masses. The Standard Model and its possible field theoretic extensions, like Grand Unified Theories (GUTs) and supersymmetric GUTs, do not provide means to calculate the fermion masses. In the context of unified theories the fermion masses are expected to arise due to some underlying Planck scale physics. Superstring theory [1] is a unique theory in the sense that it is believed to be a consistent theory of quantum gravity while at the same time consistent heterotic string vacua [2] give rise to massless spectra that closely resemble the Standard Model [3]. At present, string theory provides the best tool to probe Planck scale physics.

In the context of superstring theory one can calculate the Yukawa couplings in terms of scattering amplitudes between the string states and certain VEVs that parameterize the string vacuum [4,5]. In their low energy limit superstring theories give rise to effective $N = 1$ supergravity [6]. In the standard $N = 1$ supergravity model the electroweak Higgs VEV is fixed by the initial boundary conditions at the unification scale and their evolution to the electroweak scale by the renormalization group equations [7]. Thus, in superstring theories one may be able to calculate the fermion masses. For this purpose one must construct realistic superstring models. The construction of realistic superstring models can be pursued in several approaches. One possibility is to go through a simple [8] or a semi–simple [9,10,11,12] unifying group at intermediate energy scale. Another is to derive the Standard Model directly from string theory [13,14,15,17,16].

In Refs. [15,16,17] realistic superstring standard–like models were constructed in the four dimensional free fermionic formulation [18]. One of the important achievements of the superstring derived standard–like models in the free fermionic formulation is the possible explanation of the top quark mass hierarchy and the successful prediction of the top quark mass. In Ref. [16] the top quark mass was predicted to be in the approximate mass range

$$m_t \approx 175 - 180 \ GeV, \tag{1}$$

three years prior to its experimental observation. Remarkably, this prediction is in agreement with the top quark mass as observed by the recent CDF and D0 collaborations [19].

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The superstring standard–like models have a very important property that enhances their predictive power. There are three and only three generations in the massless spectrum [17]. There are no additional generations and mirror generations. Therefore, the identification of the three light generations is unambiguous. This property of the standard–like models enables, up to some motivated assumptions with regard to the light Higgs spectrum, unambiguous identification of the light fermion spectrum. In this paper I will focus on the calculation of the heavy fermion masses.

The free fermionic standard–like models suggest an explanation for the top quark mass hierarchy. At the cubic level of the superpotential only the top quark gets a nonvanishing mass term. The mass terms for the lighter quarks and leptons are obtained from nonrenormalizable terms. Standard Model singlet fields in these nonrenormalizable terms obtain nonvanishing VEVs by the application of the Dine–Seiberg–Witten (DSW) mechanism [20]. Thus, the order N nonrenormalizable terms, of the form $cffh(\Phi/M)^{N-3}$, become effective trilinear terms, where f, h, Φ denote fermions, electroweak scalar doublets and Standard Model scalar singlets, respectively. M is a Planck scale mass to be defined later. The effective Yukawa couplings are therefore given by $\lambda = c(\langle\Phi\rangle/M)^{N-3}$. The calculation of the coefficients c for the heavy fermion family is the main focus of the present paper.

In this paper I discuss the calculation of the heavy fermion masses in the superstring derived standard–like models. The analysis is illustrated in the toy model of Ref. [16]. In this model the top quark Yukawa coupling is obtained from a cubic level term in the superpotential while the bottom quark and the tau lepton Yukawa couplings are obtained from quartic order terms. The calculation of the cubic and quartic order correlators, is described in detail. The Standard Model singlet fields in the quartic order bottom quark and tau lepton mass terms acquire a VEV by application of the DSW mechanism. These VEVs parameterize the effective bottom quark and tau lepton Yukawa couplings. The dependence of the effective bottom quark and tau lepton Yukawa couplings on the DSW VEVs is studied. It is shown that there is substantial freedom in the resulting numerical values of the effective Yukawa couplings. This freedom in turn affects the low energy prediction of the top quark mass.

The three heavy generation Yukawa couplings are extrapolated from the unification scale to the electroweak scale by using the coupled two–loop supersymmetric renormalization group equations. Agreement with the low energy gauge parameters $\alpha_{\text{em}}(M_Z)$, $\sin^2\theta_W(M_Z)$ and $\alpha_s(M_Z)$ is imposed. This requires that some

additional vector–like matter, beyond the MSSM and which appear in the mass-less spectrum of the superstring standard–like models, exist at intermediate energy scales [22,23]. The mass scales of the additional states is imposed by hand and their derivation from the string model is left for future work. The intermediate matter thresholds also affect the evolution of the Yukawa couplings and consequently the low energy predictions of the fermion masses [25,26].

The bottom quark and W–boson masses are used to calculate the electroweak VEV ratio, $\tan\beta = v_1/v_2$. The extrapolated Yukawa couplings and $\tan\beta$ are then used to calculate the top quark mass and the ratio of the Yukawa couplings $\lambda_b(M_Z)/\lambda_\tau(M_Z)$. As the VEV in the DSW mechanism, which fixes the effective bottom quark and tau lepton Yukawa couplings is varied, the predicted top quark mass is found in the approximate range

$$90 \text{ GeV} \leq m_t(m_t) \leq 205 \text{ GeV}.$$

and $\tan\beta$ is found in the approximate range

$$0.6 \leq \tan\beta \leq 28.$$

Thus, the predicted top quark mass can exist in a wide range and is correlated with the predicted value of $\tan\beta$. For fixed values of the VEVs in the DSW mechanism the top quark mass and $\tan\beta$ are of course fixed. The b/τ mass ratio is also found to be in good agreement with experiment. It is found that the intermediate matter thresholds which are required for string gauge coupling unification also help in maintaining the correct b/τ mass ratio.

In general, $\tan\beta$ can be fixed by minimizing the Higgs potential. I examine the minimization of the Higgs potential and illustrate that the calculated $\tan\beta$ can, in principle, be compatible with the minimization of the one–loop Higgs effective potential. For this purpose, the soft SUSY breaking parameters are fixed by hand and determination of those terms in the string models is left for future work.

The paper is organized as follows. In section 2, I review the realistic free fermionic models. Section 3 summarizes the tools needed for the calculation of the Yukawa couplings. In section 4 the calculation of the top quark Yukawa coupling is presented. In section 5 and 6 the calculation of the bottom quark and tau lepton Yukawa couplings is described in detail. In section 5 the calculation of the quartic

order bottom quark and tau lepton mass terms is outlined. In section 6 the dependence of the effective bottom quark and tau lepton Yukawa couplings on the DSW VEVs is investigated. In section 7 the top, bottom and tau lepton Yukawa couplings are extrapolated to the electroweak scale, in the presence the intermediate matter thresholds, by using the coupled gauge and Yukawa two–loop RGEs. In section 8 I discuss the minimization of the one–loop Higgs effective potential and possible corrections from the supersymmetric mass spectrum. Section 9 concludes the paper.

2. Realistic free fermionic models

The free fermionic models are constructed by choosing a set of boundary condition basis vectors and one–loop GSO projection coefficients [18]. The possible boundary condition basis vectors and one-loop GSO phases are constrained by the string consistency constraints. The physical states are obtained by applying the generalized GSO projections. The physical spectrum, its symmetries and interactions are then completely determined. The low energy effective field theory is obtained by S–matrix elements between external states. The Yukawa couplings and higher order nonrenormalizable terms in the superpotential are obtained by calculating correlators between vertex operators. For a correlator to be nonvanishing all the symmetries of the model must be conserved. Thus, the boundary condition basis vectors and the one–loop GSO projection coefficients completely determine the phenomenology of the models.

The first five basis vectors in the models that I discuss consist of the NAHE set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$ [27,17]. The vector S in this set is the supersymmetry generator. The two basis vectors $\{\mathbf{1}, S\}$ produce a model with $N = 4$ space–time supersymmetry and $SO(44)$ gauge group. At this level all of the internal world–sheet fermions are equivalent. At the level of the NAHE set the gauge group is $SO(10) \times SO(6)^3 \times E_8$. The sectors b_1, b_2 and b_3 each produce sixteen spinorial 16 representation of $SO(10)$. The number of generations is reduced to three and the $SO(10)$ gauge group is broken to one of its subgroups, $SU(5) \times U(1)$, $SU(3) \times SU(2) \times U(1)^2$ or $SO(6) \times SO(4)$ by adding to the NAHE set three additional basis vectors, $\{\alpha, \beta, \gamma\}$. In the first two cases the basis vector that breaks the $SO(10)$ symmetry to $SU(5) \times U(1)$ must contain half integral boundary conditions for the world–sheet complex fermions that generate the $SO(10)$ symmetry. This basis vector is denoted as the vector γ.

The NAHE set plus the vector 2γ divide the world–sheet fermions into several groups. The six left–moving real fermions, $\chi^{1,\cdots,6}$ are paired to form three complex fermions denoted χ^{12}, χ^{34} and χ^{56}. These complex fermions produce the SUSY charges of the physical states. The sixteen right–moving complex fermions $\bar\psi^{1\cdots5}, \bar\eta^1, \bar\eta^2, \bar\eta^3, \bar\phi^{1\cdots8}$ produce the observable and hidden gauge groups, that arise from the sixteen dimensional compactified space of the heterotic string in ten dimensions. The complex world–sheet fermions, $\bar\psi^{1\cdots5}$, generate the $SO(10)$ symmetry; $\bar\phi^{1,\cdots8}$ produce the hidden E_8 gauge group; and $\bar\eta^1$, $\bar\eta^2$, $\bar\eta^3$ give rise to three horizontal $U(1)$ symmetries. Finally, the twelve left–moving, $\{y,\omega\}^{1\cdots6}$, and twelve right–moving, $\{\bar y,\bar\omega\}^{1\cdots6}$, real fermions correspond to the left/right symmetric internal conformal field theory of the heterotic string, or equivalently to the six dimensional compactified manifold in a bosonic formulation. The set of internal fermions $\{y,\omega|\bar y,\bar\omega\}^{1\cdots6}$ plays a fundamental role in the determination of the low energy properties of the realistic free fermionic models. In particular the assignment of boundary conditions, in the vector γ, to this set of internal world–sheet fermions selects cubic level Yukawa couplings for $+2/3$ or $-1/3$ charged quarks.

The three boundary condition basis vectors $\{\alpha,\beta,\gamma\}$ break the observable $SO(10)$ gauge group to one of its subgroups. At the same time the horizontal symmetries are broken to factors of $U(1)'s$. Three $U(1)$ symmetries arise from the complex right–moving fermions $\bar\eta^1$, $\bar\eta^2$, $\bar\eta^3$. Additional horizontal $U(1)$ symmetries arise by pairing two of the right–moving real internal fermions $\{\bar y,\bar\omega\}$. For every right–moving $U(1)$ symmetry, there is a corresponding left–moving global $U(1)$ symmetry that is obtained by pairing two of the left–moving real fermions $\{y,\omega\}$. Each of the remaining world–sheet left–moving real fermions from the set $\{y,\omega\}$ is paired with a right–moving real fermion from the set $\{\bar y,\bar\omega\}$ to form a Ising model operator.

I now turn to describe the properties of the toy model of Ref. [16], which are important for the calculation of the heavy fermion masses. The three additional boundary condition basis vectors, beyond the NAHE set, in the model of Ref. [16] are given in table 1. In this toy model an additional complication arises due to the appearance of additional space–time vector bosons from twisted sectors [28]. A combination of the $U(1)$ symmetries is enhanced to $SU(2)$. The weak hypercharge then arises as a combination of the diagonal generator of the custodial $SU(2)$ gauge group and the other $U(1)$ generators. The custodial $SU(2)$ symmetry can be broken, near the Planck scale, by a VEV of the custodial $SU(2)$ doublets, along F and D flat directions. I will assume the existence of such a solution and neglect the effect of the custodial $SU(2)$ symmetry. I will therefore focus on the part of

the gauge group that arises solely from the untwisted sector and therefore on the properties that are common to a large class of free fermionic models [17]. The reason for illustrating the calculation in the toy model of Ref. [16] is because in this model nonvanishing bottom quark and tau lepton mass terms arise at the quartic order of the superpotential whereas, for example, in the model of Ref. [15] such terms only appear at the quintic order.

In the models of Refs. [15,16] the complex right–moving fermions $\bar{\psi}^{1,\cdots,5}$ produce the generators of the $SU(3) \times SU(2) \times U(1)_C \times U(1)_L$ gauge group. The right–moving complex fermions $\bar{\eta}^{1,2,3}$ generate three $U(1)$ currents denoted by $U(1)_{r_{1,2,3}}$. Three additional right–moving $U(1)$ symmetries, denoted $U(1)_{r_j}$ (j=4,5,6), arise from three additional complexified right–moving fermions from the set $\{\bar{y}, \bar{\omega}\}$ denoted by

$$e^{i\bar{\zeta}_1} = \frac{1}{\sqrt{2}}(\bar{y}^3 + i\bar{y}^6), \tag{2a}$$

$$e^{i\bar{\zeta}_2} = \frac{1}{\sqrt{2}}(\bar{y}^1 + i\bar{\omega}^5), \tag{2b}$$

$$e^{i\bar{\zeta}_3} = \frac{1}{\sqrt{2}}(\bar{\omega}^2 + i\bar{\omega}^4). \tag{2c}$$

For every local right–moving $U(1)_r$ symmetry there is a corresponding global left–moving $U(1)_\ell$ symmetry. The first three, denoted $U(1)_{\ell_j}$ ($j = 1, 2, 3$), correspond to the charges of the supersymmetry generator χ^{12}, χ^{34} and χ^{56}, respectively. The last three, denoted $U(1)_{\ell_j}$ ($j = 4, 5, 6$), arise from the three additional complexified left–moving fermions from the set $\{y, \omega\}$ denoted by

$$e^{i\zeta_1} = \frac{1}{\sqrt{2}}(y^3 + iy^6), \tag{3a}$$

$$e^{i\zeta_2} = \frac{1}{\sqrt{2}}(y^1 + i\omega^5), \tag{3b}$$

$$e^{i\zeta_3} = \frac{1}{\sqrt{2}}(\omega^2 + i\omega^4). \tag{3c}$$

Finally, in the models of Refs. [15, 16] there are six Ising model operators denoted by

$$\sigma^i = \{\omega^1\bar{\omega}^1, y^2\bar{y}^2, \omega^3\bar{\omega}^3, y^4\bar{y}^4, y^5\bar{y}^5, \omega^6\bar{\omega}^6\}, \tag{4}$$

which are obtained by pairing a left–moving real fermion with a right–moving real fermion.

The full massless spectrum of this model is given in Ref. [28]. Here I list only the states that are relevant for the analysis of the heavy fermion mass terms. The sectors b_1, b_2 and b_3 produce three chiral generations, $G_\alpha = e^c_{L_\alpha} + u^c_{L_\alpha} + N^c_{L_\alpha} + d^c_{L_\alpha} + Q_\alpha + L_\alpha$ ($\alpha = 1, \cdots, 3$), with charges under the horizontal symmetries. For every generation, G_j there are two right–moving, $U(1)_{r_j}$ and $U(1)_{r_{j+3}}$, symmetries. For every right–moving $U(1)$ gauged symmetry, there is a corresponding left–moving global $U(1)$ symmetry, $U(1)_{\ell_j}$ and $U(1)_{\ell_{j+3}}$. Each sector b_1, b_2 and b_3 has two Ising model operators, (σ_4, σ_5), (σ_2, σ_6) and (σ_1, σ_3), respectively, obtained by pairing a left–handed real fermion with a right–handed real fermion. In the superstring de-rived standard–like models the vectors b_1, b_2, b_3 are the only vectors in the additive group Ξ which give rise to spinorial 16 representation of $SO(10)$. This property enhances the predictability of the superstring derived standard–like models.

The Neveu–Schwarz (NS) sector corresponds to the untwisted sector of the orbifold model and produces in addition to the gravity and gauge multiplets three pairs of electroweak scalar doublets $\{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}$, three pairs of $SO(10)$ singlets with $U(1)$ charges, $\{\Phi_{12}, \Phi_{23}, \Phi_{13}, \bar{\Phi}_{12}, \bar{\Phi}_{23}, \bar{\Phi}_{13}\}$, and three singlets, which are singlets of the entire four dimensional gauge group, ξ_1, ξ_2, ξ_3.

The sector $S + b_1 + b_2 + \alpha + \beta$ ($\alpha\beta$ sector) also produces states that transform only under the observable gauge group. In addition to two pairs of electroweak doublets, $\{h_{45}, \bar{h}_{45}, h'_{45}, \bar{h}'_{45}\}$, there are four pairs of $SO(10)$ singlets with horizontal $U(1)$ charges, $\{\Phi_{45}, \bar{\Phi}_{45}, \Phi'_{45}, \bar{\Phi}'_{45}, \Phi_{1,2,}, \bar{\Phi}_{1,2}\}$.

The spectrum described above is generic to a large class of superstring standard–like models that utilize the NAHE set of basis vectors. The states from the Neveu–Schwarz sector and the sectors b_1, b_2 and b_3 are the states which arise from the underlying $Z_2 \times Z_2$ orbifold compactification. These states are there-fore common to all the superstring standard–like models that use the NAHE set. Different models mainly differ by the assignment of boundary conditions to the set of internal fermions $\{y, \omega | \bar{y}, \bar{\omega}\}$ in the basis vectors beyond the NAHE set. Consequently, the observable spectrum in different models differs by the values of horizontal charges. A vector combination of the form $b_1 + b_2 + \alpha + \beta$ is also common in the free fermionic models that use the NAHE set. The states from this sector are important in the free fermionic standard–like models for generating the fermion mass hierarchy and for producing flat directions. Therefore, the results discussed in this paper are shared by a large class of free fermionic standard–like models.

3. Tools for calculating the fermion mass terms

Here I summarize the well known tools needed for the analysis of the nonrenormalizable terms. Further details on the derivation of these rules are given in ref. [5]. Renormalizable and nonrenormalizable contributions to the superpotential are obtained by calculating correlators between vertex operators

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle, \tag{5}$$

where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators. The vertex operators that appear in the fermion mass terms have the following generic form,

$$V_{(q)}^\ell = e^{(qc)} \, \mathcal{L}^\ell \, e^{(i\alpha\chi_{12})} \, e^{(i\beta\chi_{34})} \, e^{(i\gamma\chi_{56})}$$

$$\left(\prod_j e^{(iq_i\zeta_j)} \; \{\sigma's\} \prod_j e^{(i\bar{q}_i\bar{\zeta}_j)} \right)$$

$$e^{(i\bar{\alpha}\bar{\eta}_1)} \, e^{(i\bar{\beta}\bar{\eta}_2)} \, e^{(i\bar{\gamma}\bar{\eta}_3)} \, e^{(iW_R \cdot \bar{J})}$$

$$e^{(i\frac{1}{2}KX)} \, e^{(i\frac{1}{2}K \cdot \bar{X})} \tag{6}$$

where,

- $e^{(qc)}$ is the ghost charge, with conformal dimension

$$h = -\frac{q^2}{2} - q. \tag{7}$$

In the canonical picture $q = -1/2$ for fermions and $q = -1$ for bosons.

- $\mathcal{L}^\ell$ is the Lorentz group factor and signals the space–time spin of a state. The space–time spin of a state is determined by the boundary condition of the world–sheet ψ^μ field. A periodic ψ^μ produces the spinor representation of the Lorentz group and is represented by the conformal field S_α, where α is the space–time spinor index. An antiperiodic ψ^μ produces space–time bosons, denoted ψ^μ for vectors and I for scalars. The conformal dimensions of these fields are,

$$I \qquad (0,0) \tag{8a}$$

$$S_\alpha \qquad (\tfrac{1}{4}, 0) \tag{8b}$$

$$\psi^\mu \qquad (\tfrac{1}{2}, 0) \tag{8c}$$

respectively.

- e^{iqf} and $e^{i\bar{q}\bar{f}}$ are the factors that arise from complexified fermions, which produce global left–moving and local right–moving $U(1)$ currents, respectively. A pair of left–moving (or right–moving) real fermions f_1, f_2 which are complexified,

$$f = \frac{1}{\sqrt{2}}(f_1 + if_2) = e^{-iH} \qquad , \qquad f^* = \frac{1}{\sqrt{2}}(f_1 - if_2) = e^{iH} \tag{9}$$

produce a $U(1)$ current with charges,

$$Q(f) = \frac{1}{2}\alpha(f) + F(f), \tag{10}$$

where $\alpha(f)$ and $F(f)$ are the boundary condition and fermion number of the complex world–sheet fermion f. The conformal dimension of a complex fermion is given by $h = q^2/2$ and $\bar{h} = \bar{q}^2/2$.

- $\sigma's$: A left–moving real fermion, f, which is paired with a right–moving real fermion $\bar{f}$, produces an Ising model operator with the following conformal fields,

$$I \qquad (0,0), \tag{11a}$$

$$\sigma_\pm(z, \bar{z}) \qquad (\tfrac{1}{16}, \tfrac{1}{16}), \tag{11b}$$

$$f(z) \qquad (\tfrac{1}{2}, 0), \tag{11c}$$

$$f(\bar{z}) \qquad (0, \tfrac{1}{2}), \tag{11d}$$

$$\epsilon \equiv f(z)f(\bar{z}) \qquad (\tfrac{1}{2}, \tfrac{1}{2}), \tag{11e}$$

where $\sigma_\pm$ are the order and disorder operators and ϵ is the energy operator. The order and disorder operators arise when both f and $\bar{f}$ are periodic in a given sector α. The remaining fields arise when none, left or right, or both left and right fermion oscillators act on the vacuum.

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- $\mathrm{e}^{(iW_R \cdot \bar{J})}$ is the factor that arises due to the right–moving non–Abelian gauge group. The conformal dimension is given by $\bar{h} = W \cdot W/2$ where W is the weight vector of a representation R.

- $\mathrm{e}^{(i\frac{1}{2}KX)}$ and $\mathrm{e}^{(i\frac{1}{2}K\cdot\bar{X})}$ arise from the Poincare quantum numbers.

For the massless states, the conformal dimension $h = 1$ and $\bar{h} = 1$. An important check on the normalization of the various $U(1)$ factors is that indeed $h = 1$ and $\bar{h} = 1$ for the vertex operators of the massless states.

The first step in calculating the fermion masses is extracting the possible non vanishing correlators. This is achieved by imposing invariance under all the local Abelian and non Abelian local gauge symmetries and the other string selection rules that will be discussed below. In order to verify that a potential order N mass term is indeed nonvanishing and to extract quantitative results from the string derived models one must calculate the order N correlators. The second step is therefore the actual calculation of the potentially nonvanishing correlators.

The tri–level string amplitude is given by

$$A_N = \frac{g^{N-2}}{(2\pi)^{N-3}}\mathcal{N}\int\prod_{i=1}^{N-3}d^2 z_i\langle V_1^f(z_\infty)V_2^f(1)V_3^b(z_1)\cdots V_{N_1}^b(z_{N-3})V_N^b(0)\rangle, \quad (12)$$

where $\mathcal{N} = \sqrt{2}$ is a normalization factor and SL(2,C) invariance is used to fix the location of three of the vertex operators at $z = z_\infty, 1, 0$. For a correlator to be nonvanishing all the symmetries of the model must be conserved. Also for tree–level amplitudes the total ghost charge must be -2. Since a bosonic (fermionic) vertex operator in the canonical picture carries ghost charge -1 ($-1/2$), picture changing is required for $N \geq 4$ amplitudes. To obtain the correct ghost charge some of the vertex operators are picture changed by taking

$$V_{q+1}(z) = \lim_{w \to z} \mathrm{e}^c(w)T_F(w)V_q(z), \quad (13)$$

where T_F is the super current and in the fermionic construction is given by

$$T_F = \psi^\mu \partial_\mu X + i\sum_{I=1}^{6}\chi_I y_I \omega_I = T_F^0 + T_F^{-1} + T_F^{+1} \quad (14)$$

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with

$$T_F^{-1} = e^{-i\chi^{12}}\tau_{12} + e^{-i\chi^{34}}\tau_{34} + e^{-i\chi^{56}}\tau_{56} \quad ; \quad T_F^{-1} = (T_F^{+1})^* \tag{15}$$

where

$$\tau_{ij} = \frac{i}{\sqrt{2}}(y^i\omega^i + iy^j\omega^j) \tag{16}$$

and

$$e^{\chi^{ij}} = \frac{1}{\sqrt{2}}(\chi^i + i\chi^j). \tag{17}$$

In the models of Refs. [15,16] the complexified left–moving fermions are $y^1\omega^5$, $\omega^2\omega^4$ and y^3y^6. Thus, one of the fermionic states in every term $y^i\omega^i$ $(i = 1, ..., 6)$ is complexified and therefore can be written, for example for y^3 and y^6, as

$$y^3 = \frac{1}{\sqrt{2}}(e^{i\zeta_1} + e^{-i\zeta_1}) \quad , \quad y^6 = \frac{1}{\sqrt{2}i}(e^{i\zeta_1} - e^{-i\zeta_1}). \tag{18}$$

Consequently, every picture changing operation changes the total $U(1)_\ell = U(1)_{\ell_4} + U(1)_{\ell_5} + U(1)_{\ell_6}$ charge by ± 1. An odd (even) order term requires an even (odd) number of picture changing operations to get the correct ghost number [5]. Thus, for A_N to be non vanishing, the total $U(1)_\ell$ charge, before picture changing, has to be an odd (even) number, for even (odd) order terms, respectively. Similarly, in every pair $y_i\omega_i$, one real fermion, either y_i or ω_i, remains real and is paired with the corresponding right–moving real fermion to produce an Ising model sigma operator. Every picture changing operation changes the number of left–moving real fermions by one. This property of the standard–like models significantly reduces the number of potential non vanishing terms.

The following Operator Product Expansions (OPEs) are used in the evaluation of the fermion mass terms

- Ghosts

$$\langle e^{(-c/2)}(z_1)e^{(-c/2)}(z_2)e^{(-c)}(z_3)\rangle = z_{12}^{-1/4}z_{13}^{-1/2}z_{23}^{-1/2} \tag{19}$$

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- Lorentz group

$$\langle S_\alpha(z_1) S_\beta(z_2) \rangle = C_{\alpha\beta} z_{12}^{-1/2} \tag{20}$$

- Correlator of exponentials

$$\langle \prod_j e^{i\vec{\alpha}_j \cdot \vec{J}} \rangle = \prod_{i<j} (z_{ij})^{\vec{\alpha}_i \cdot \vec{\alpha}_j} \tag{21}$$

- Ising model correlators [29,30],

$$\langle f(z_1) \sigma^\pm(z_2) \rangle = \frac{1}{\sqrt{2}} z_{12}^{-1/2} \sigma^\mp(z_1) \tag{22a}$$

$$\langle \sigma^\pm(z_1) \sigma^\pm(z_2) \rangle = z_{12}^{-1/8} (\bar{z}_{12})^{-1/8} \tag{22b}$$

$$\langle \sigma^+(z_1) \sigma^-(z_2) f(z_3) \rangle = \frac{1}{\sqrt{2}} z_{12}^{3/8} (\bar{z}_{12})^{-1/8} (z_{13} z_{23})^{-1/2} \tag{22c}$$

$$\langle \sigma^+(z_1) \sigma^-(z_2) \bar{f}(z_3) \rangle = \frac{1}{\sqrt{2}} z_{12}^{-1/8} (\bar{z}_{12})^{3/8} (\bar{z}_{13} \bar{z}_{23})^{-1/2} \tag{22d}$$

$$\langle \sigma_\pm(z_\infty) \sigma_\pm(1) \sigma_\pm(z) \sigma_\pm(0) \rangle = \frac{1}{\sqrt{2}} |z_\infty|^{-1/4} |1-z|^{-1/4} |z|^{-1/4} (1 + |z| + |1-z|)^{1/2} \tag{22e}$$

where

$$z_{ij} = z_i - z_j. \tag{23}$$

4. Calculation of the Top quark Yukawa coupling

The superstring derived standard–like models suggest a superstring mechanism which explains the suppression of the lighter quark and lepton masses relative to the top quark mass. These models suggest that only the top quark gets a nonvanishing cubic level mass term while the lighter quarks and leptons get their mass terms from nonrenormalizable terms which are suppressed relative to the leading cubic level terms.

The assignment of boundary conditions in the basis vector γ for the internal world–sheet fermions, $\{y, \omega | \bar{y}, \bar{\omega}\}$ selects a cubic level mass term for $+2/3$ or $-1/3$ charged quarks. For each of the sectors b_1, b_2 and b_3 the fermionic boundary conditions selects the cubic level Yukawa couplings according to the difference,

$$\Delta_j = |\gamma(L_j) - \gamma(R_j)| = 0, 1 \tag{24}$$

where $\gamma(L_j)/\gamma(R_j)$ are the boundary conditions in the vector γ for the internal world–sheet fermions from the set $\{y, \omega | \bar{y}, \bar{\omega}\}$, that are periodic in the vector b_j. If

$$\Delta_j = 1 \tag{25}$$

then a Yukawa coupling for the $+2/3$ charged quark from the sector b_j is nonzero and the Yukawa coupling for the $-1/3$ charged quark vanishes. The opposite occurs if $\Delta_j = 0$. Thus, the states from each of the sectors b_1, b_2 and b_3 can have a cubic level Yukawa coupling for the $+2/3$ or $-1/3$ charged quark, but not for both. We can construct string models in which both $+2/3$ and $-1/3$ charged quarks get a cubic level mass term. The model of table 2 is an example of such a model. In this model,

$$\Delta_1 = |\gamma(y^3 y^6) - \gamma(\bar{y}^3 \bar{y}^6)| = 1, \tag{26a}$$
$$\Delta_2 = |\gamma(y^1 \omega^6) - \gamma(\bar{y}^1 \bar{\omega}^6)| = 0, \tag{26b}$$
$$\Delta_3 = |\gamma(\omega^1 \omega^3) - \gamma(\bar{\omega}^1 \bar{\omega}^3)| = 0. \tag{26c}$$

Consequently, in this model there is a cubic level mass term for the $+2/3$ charged quark from the sector b_1 and cubic level mass terms for the $-1/3$ charged quark and for charged leptons from the sectors b_2 and b_3.

We can also construct string models in which only $+2/3$ charged quarks get a nonvanishing cubic level mass term. The model of table 1 is an example of such a model. In this model

$$\Delta_1 = |\gamma(y^3 y^6) - \gamma(\bar{y}^3 \bar{y}^6)| = 1, \tag{27a}$$
$$\Delta_2 = |\gamma(y^1 \omega^5) - \gamma(\bar{y}^1 \bar{\omega}^5)| = 1, \tag{27b}$$
$$\Delta_3 = |\gamma(\omega^2 \omega^4) - \gamma(\bar{\omega}^2 \bar{\omega}^4)| = 1. \tag{27c}$$

Therefore, in this model $\Delta_1 = \Delta_2 = \Delta_3 = 1$ and cubic level mass terms are obtained for the $+2/3$ charged quarks from the sectors b_1, b_2 and b_3.

In Ref. [31] the superstring up/down selection rule is proven by using the string consistency constraints and Eq. (18) to show that either the $+2/3$ or the $-1/3$ mass term is invariant under the $U(1)_j$ symmetry.

In the model of Ref. [16] the following terms are obtained in the observable sector at the cubic level of the superpotential

$$
\begin{aligned}
W_3 = \{ & (u^c_{L_1} Q_1 \bar{h}_1 + N^c_{L_1} L_1 \bar{h}_1 + u^c_{L_2} Q_2 \bar{h}_2 + N^c_{L_2} L_2 \bar{h}_2 + u^c_{L_3} Q_3 \bar{h}_3 + N^c_{L_3} L_3 \bar{h}_3) \\
& + h_1 \bar{h}_2 \bar{\Phi}_{12} + h_1 \bar{h}_3 \bar{\Phi}_{13} + h_2 \bar{h}_3 \bar{\Phi}_{23} + \bar{h}_1 h_2 \Phi_{12} + \bar{h}_1 h_3 \Phi_{13} + \bar{h}_2 h_3 \Phi_{23} \\
& + \Phi_{23} \bar{\Phi}_{13} \Phi_{12} + \bar{\Phi}_{23} \Phi_{13} \bar{\Phi}_{12} + \bar{\Phi}_{12} (\bar{\Phi}_1 \Phi_1 + \bar{\Phi}_2 \Phi_2) + \Phi_{12} (\Phi_1 \bar{\Phi}_1 + \Phi_2 \bar{\Phi}_2) \\
& + \frac{1}{2} \xi_3 (\Phi_{45} \bar{\Phi}_{45} + h_{45} \bar{h}_{45} + \Phi'_{45} \bar{\Phi}'_{45} + h'_{45} \bar{h}'_{45} + \Phi_1 \bar{\Phi}_1 + \Phi_2 \bar{\Phi}_2) \\
& + h_3 \bar{h}_{45} \bar{\Phi}'_{45} + \bar{h}_3 h_{45} \Phi'_{45} + h_3 \bar{h}'_{45} \Phi_{45} + \bar{h}_3 h'_{45} \bar{\Phi}_{45} \\
& + \frac{1}{2} (\xi_1 D_1 \bar{D}_1 + \xi_2 D_2 \bar{D}_2) + \frac{1}{\sqrt{2}} (D_1 \bar{D}_2 \phi_2 + \bar{D}_1 D_2 \bar{\phi}_1) \},
\end{aligned}
\tag{28}
$$

At the cubic level of the superpotential the $+2/3$ charged quarks get nonvanishing mass terms,

$$
u^c_{L_1} Q_1 \bar{h}_1 \;+\; u^c_{L_2} Q_2 \bar{h}_2 \;+\; u^c_{L_3} Q_3 \bar{h}_3,
\tag{29}
$$

while the $-1/3$ charged quarks and the charged leptons cubic level mass terms vanish. This selection mechanism results from the specific assignment of boundary conditions that specify the string models, with $\Delta_j = 1$ for $(j = 1, 2, 3)$. Any free fermionic standard–like model or flipped $SU(5)$ model (i.e. that uses the vector γ with $1/2$ boundary conditions), which satisfies the condition $\Delta_j = 1$ for $(j = 1, 2, 3)$ will therefore have cubic level mass terms only for $+2/3$ charged quarks.

Due to the horizontal, $U(1)_{r_j}$, symmetries of the string models, each of the chiral generations, from the sectors b_j, $j = 1, 2, 3$, can couple at the cubic level only to one of the Higgs pairs h_j, $\bar{h}_j$. This results due to the fact that the states from a sector b_j and the Higgs doublets h_j and $\bar{h}_j$ are charged with respect to one of the horizontal $U(1)_j$, $j = 1, 2, 3$ symmetries. Analysis of the renormalizable and nonrenormalizable Higgs mass terms suggests that for some appropriate choices of flat F and D directions, only one pair of the Higgs doublets remains light at low energies [32]. In the flipped $SU(5)$ string model and the standard–like models, it has been found that we must impose [33,15,16,34],

$$
\langle \Phi_{12}, \bar{\Phi}_{12} \rangle = 0,
\tag{30}
$$

and that Φ_{45}, and $\bar{\Phi}_{13}$ or $\bar{\Phi}_{23}$, must be different from zero. From this result and

the cubic level superpotential it follows that in any flat F and D solution, h_3 and $\bar{h}_3$ obtain a Planck scale mass. This result is a consequence of the symmetry of the vectors α and β with respect to the vectors b_1 and b_2. At the level of the NAHE set there is a cyclic symmetry between the sectors b_1, b_2 and b_3. The sectors α and β break the cyclic symmetry. The consequence is that h_3 and $\bar{h}_3$ do not contribute to the light Higgs representations and obtain superheavy mass from cubic level superpotential terms. At this level a residual Z_2 symmetry exist between the sectors b_1 and b_2 and is broken further by the choices of flat directions. Higher order nonrenormalizable terms then give superheavy mass to $\bar{h}_1$ or $\bar{h}_2$ [34]. As a result only one nonvanishing mass term, namely the top quark mass term, remains at low energies. It should be emphasized that the detailed analysis of the Higgs mass spectrum in the superstring standard–like models was performed in the model of Ref. [15]. However, the observable massless spectrum in the models of Ref. [16,22] is similar to that to the the model of Ref. [15], with slight variations in the charges under the horizontal charges. The models differ by the assignment of boundary conditions in the basis vectors $\{\alpha, \beta, \gamma\}$, which affects mainly the spectrum under the hidden sector and the horizontal charges. Consequently, it is expected that similar results with regard to the light Higgs spectrum can be found in the models of ref. [16,22]. I therefore assume the existence of a solution with $\bar{h}_2$ as one of the light Higgs multiplets, in which case the top quark mass term is

$$u_2 Q_2 \bar{h}_2.$$

The coefficients of the cubic–level terms in the superpotential, $\int d^2\theta \Phi_1 \Phi_2 \Phi_3$ are given by Eq. (12) with $N = 3$,

$$A_3 = g\sqrt{2}\langle V_{1(-1/2)}^f(z_1) V_{2(-1/2)}^f(z_2) V_{3(-1)}^b(z_3)\rangle \tag{31}$$

The vertex operators in the canonical picture in the top quark mass term, $u_2 Q_2 \bar{h}_2$, are

$$u_{2(-\frac{1}{2})}^f = e^{(-\frac{1}{2}c)} S_\alpha e^{(i\frac{1}{2}\chi_{34})} e^{(i\frac{1}{2}\zeta_2)} \sigma_2^+ \sigma_6^+ e^{(i\frac{1}{2}\bar{\zeta}_2)} e^{(i\frac{1}{2}\bar{\eta}_2)} e^{(i\bar{J}_{16}\cdot\bar{W}_{16})} e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

$$Q_{2(-\frac{1}{2})}^f = e^{(-\frac{1}{2}c)} S_\beta e^{(i\frac{1}{2}\chi_{34})} e^{(-i\frac{1}{2}\zeta_2)} \sigma_2^+ \sigma_6^+ e^{(-i\frac{1}{2}\bar{\zeta}_2)} e^{(i\frac{1}{2}\bar{\eta}_2)} e^{(i\bar{J}_{16}\cdot\bar{W}_{16})} e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

$$\bar{h}_{2(-1)}^b = e^{(-c)} e^{(-i\chi_{34})} e^{(iW_{10}\cdot J_{10})} e^{(-i\bar{\eta}_2)} e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})}, \tag{32}$$

The cubic level amplitude is given by

$$
\begin{aligned}
A_3 = g\sqrt{2}\Big\{ & \langle \mathrm{e}^{(-c/2)}(z_1)\mathrm{e}^{(-c/2)}(z_2)\mathrm{e}^{(-c)}(z_3)\rangle \\
& \langle S_\alpha(z_1)S_\beta(z_2)\rangle \\
& \langle \mathrm{e}^{(\mathrm{i}\frac{1}{2}\chi_{34})}(z_1)\mathrm{e}^{(\mathrm{i}\frac{1}{2}\chi_{34})}(z_2)\mathrm{e}^{(-\mathrm{i}\chi_{34})}(z_3)\rangle \\
& \langle \mathrm{e}^{(\mathrm{i}\frac{1}{2}\zeta_2)}(z_1)\mathrm{e}^{(-\mathrm{i}\frac{1}{2}\zeta_2)}(z_2)\rangle \\
& \langle \sigma_2^+(z_1)\sigma_2^+(z_2)\rangle \\
& \langle \sigma_6^+(z_1)\sigma_6^+(z_2)\rangle \\
& \langle \mathrm{e}^{(\mathrm{i}\frac{1}{2}\bar\zeta_2)}(z_1)\mathrm{e}^{(-\mathrm{i}\frac{1}{2}\bar\zeta_2)}(z_2)\rangle \\
& \langle \mathrm{e}^{(\mathrm{i}\bar J_{\bar 3,1}\cdot\bar W_{\bar 3,1})}(z_1)\mathrm{e}^{(\mathrm{i}\bar J_{\bar 3,2}\cdot\bar W_{3,2})}(z_2)\mathrm{e}^{(\mathrm{i}\bar J_{1,2}\cdot\bar W_{1,2})}(z_3)\rangle \\
& \langle \mathrm{e}^{(\mathrm{i}\frac{1}{2}\bar\eta_2)}(z_1)\mathrm{e}^{(\mathrm{i}\frac{1}{2}\bar\eta_2)}(z_2)\mathrm{e}^{(-\mathrm{i}\bar\eta_2)}(z_3)\rangle \\
& \left\langle \prod_{i=1}^{4}\mathrm{e}^{[\mathrm{i}\frac{1}{2}K_i X(i)]}\mathrm{e}^{[\mathrm{i}\frac{1}{2}K_i \bar X(i)]}\right\rangle \Big\} .
\end{aligned}
\tag{33}
$$

The correlators are evaluated using the formula given in Eqs. (19-23). Since $K_1 + K_2 + K_3 = 0$ and $K_1^2 = K_2^2 = K_3^2 = 0$, it follows that $K_1\cdot K_2 = K_1\cdot K_3 = K_2\cdot K_3 = 0$. Consequently, evaluation of the correlator in Eq. (33) yields,

$$
A_3 = g\sqrt{2}
\tag{34}
$$

which is taken as the top quark Yukawa coupling at the string unification scale.

5. Calculation of the bottom quark and tau lepton mass terms

In free fermionic standard–like (and flipped $SU(5)$) models with $\Delta_{1,2,3} = 1$, only $+2/3$ charged quarks obtain potential mass terms at the cubic level of the superpotential. There are no potential cubic level mass terms for $-1/3$ charged quarks and for charged leptons. A realistic string model must give rise to such mass terms. Consequently, in this class of models, $-1/3$ charged quarks and charged leptons must get their mass terms from nonrenormalizable terms in the superpotential. The nonrenormalizable terms have the general form

$$
c f_i f_j h (\phi/M)^{n-3}
\tag{35}
$$

where c are the calculable coefficients of the nth order correlators, f_i, f_j are the quark and lepton fields, h are the light Higgs representations and ϕ are Standard

Model singlets in the massless spectrum of the string models. The scale M is related to the Planck scale, and numerically $M \sim 1.2 \cdot 10^{18} GeV$.

In the models of Ref. [15,16], due to the up/down Yukawa superstring selection mechanism there are no potential mass terms for $-1/3$ charged quarks and for charged leptons at the cubic level of the superpotential. Such mass terms may arise from quartic, quintic or higher order terms in the superpotential. In the model of Ref. [15], for example, because of the global $U(1)_{\ell_{j+3}}$ symmetries, there are no potential bottom quark and tau lepton mass terms at the quartic order of the superpotential [31]. In this model, $Q_{\ell_{j+3}}(Q_j, L_j) = -Q_{\ell_{j+3}}(d_j, e_j)$. Consequently, the total U_ℓ charge before picture changing vanishes and the quartic order down quark and tau lepton mass terms vanish. In the model of Ref. [15] such potential non vanishing mass terms arise at the quintic order of the superpotential (in the notation of Ref. [15]),

$$W_5 = \{d^c_{L_1} Q_1 h_{45} \Phi_1^- \xi_2 + e^c_{L_1} L_1 h_{45} \Phi_1^+ \xi_2 + d^c_{L_2} Q_2 h_{45} \Phi_2^- \xi_1 + e^c_{L_2} L_2 h_{45} \bar{\Phi}_2^- \xi_1\}. \quad (36)$$

The evaluation of the coefficient of the quintic order terms involves a two dimensional complex integration. Considerable simplification will be provided if we can construct a model in which potential non–vanishing bottom quark and tau lepton mass terms are obtained from quartic order terms. Such a model was constructed in Ref. [16]. In this model $Q_{\ell_{j+3}}(Q_j, L_j) = +Q_{\ell_{j+3}}(d_j, e_j)$. Consequently, the total U_ℓ charge before picture changing is ± 1. In the model of Ref. [16], the following non vanishing mass terms for $-1/3$ charged quarks and for charged leptons are obtained at the quartic order,

$$W_4 = \{d^c_{L_1} Q_1 h'_{45} \Phi_1 + e^c_{L_1} L_1 h'_{45} \Phi_1 + d^c_{L_2} Q_2 h'_{45} \bar{\Phi}_2 + e^c_{L_2} L_2 h'_{45} \bar{\Phi}_2\}. \quad (37)$$

This quartic order terms can therefore be potential mass terms for the bottom quark and tau lepton. To evaluate the bottom quark and tau lepton masses we must first evaluate the coefficients of the quartic order correlators. The Standard Model singlet in the quartic order terms can then get a VEV, which then results in effective bottom quark and tau lepton Yukawa couplings. From Eq. (37) it is seen that if $\bar{\Phi}_2 >> \Phi_1$ then the last two terms in Eq. (37) are the bottom quark and tau lepton mass terms.

For the bottom quark mass term, the vertex operators in the canonical picture are given by

$$d^f_{2(-\frac{1}{2})} = e^{(-\frac{1}{2}c)} \, S_\alpha \, e^{(i\frac{1}{2}\chi_{34})} \, e^{(-i\frac{1}{2}\zeta_2)} \, \sigma_2^+ \sigma_6^- \, e^{(i\frac{1}{2}\bar{\zeta}_2)} \, e^{(i\frac{1}{2}\bar{\eta}_2)} \, e^{(i\bar{J}_{16} \cdot \bar{W}_{16})} \, e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

$$Q^f_{2(-\frac{1}{2})} = e^{(-\frac{1}{2}c)} \, S_\beta \, e^{(i\frac{1}{2}\chi_{34})} \, e^{(-i\frac{1}{2}\zeta_2)} \, \sigma^+_2 \sigma^+_6 \, e^{(-i\frac{1}{2}\bar{\zeta}_2)} \, e^{(i\frac{1}{2}\bar{\eta}_2)} \, e^{(i\bar{J}_{16}\cdot\bar{W}_{16})} \, e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

$$h'^b_{45(-1)} = e^{(-c)} \, e^{(-i\frac{1}{2}\chi_{12})} e^{(-i\frac{1}{2}\chi_{34})} \, \sigma^+_1 \sigma^+_2 \sigma^+_3 \sigma^+_4 \, e^{(-i\frac{1}{2}\bar{\eta}_1)} e^{(-i\frac{1}{2}\bar{\eta}_2)} \, e^{(i\bar{J}_{10}\cdot\bar{W}_{10})} \, e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

$$\bar{\Phi}^b_{2(-1)} = e^{(-c)} \, e^{(-i\frac{1}{2}\chi_{12})} e^{(-i\frac{1}{2}\chi_{34})} \, \sigma^-_1 \sigma^+_2 \sigma^+_3 \sigma^+_4 \bar{\omega}^6 \, e^{(i\frac{1}{2}\bar{\eta}_1)} \, e^{(-i\frac{1}{2}\bar{\eta}_2)} \, e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

and similarly for the tau lepton mass term,

$$e^f_{2(-\frac{1}{2})} = e^{(-\frac{1}{2}c)} \, S_\alpha \, e^{(i\frac{1}{2}\chi_{34})} \, e^{(i\frac{1}{2}\zeta_2)} \, \sigma^+_2 \sigma^+_6 \, e^{(i\frac{1}{2}\bar{\zeta}_2)} \, e^{(i\frac{1}{2}\bar{\eta}_2)} \, e^{(i\bar{J}_{16}\cdot\bar{W}_{16})} \, e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

$$L^f_{2(-\frac{1}{2})} = e^{(-\frac{1}{2}c)} \, S_\beta \, e^{(i\frac{1}{2}\chi_{34})} \, e^{(i\frac{1}{2}\zeta_2)} \, \sigma^+_2 \sigma^-_6 \, e^{(-i\frac{1}{2}\bar{\zeta}_2)} \, e^{(i\frac{1}{2}\bar{\eta}_2)} \, e^{(i\bar{J}_{16}\cdot\bar{W}_{16})} \, e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

$$h'^b_{45(-1)} = e^{(-c)} \, e^{(-i\frac{1}{2}\chi_{12})} e^{(-i\frac{1}{2}\chi_{34})} \, \sigma^+_1 \sigma^+_2 \sigma^+_3 \sigma^+_4 \, e^{(-i\frac{1}{2}\bar{\eta}_1)} e^{(-i\frac{1}{2}\bar{\eta}_2)} \, e^{(i\bar{J}_{10}\cdot\bar{W}_{10})} \, e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

$$\bar{\Phi}^b_{2(-1)} = e^{(-c)} \, e^{(-i\frac{1}{2}\chi_{12})} e^{(-i\frac{1}{2}\chi_{34})} \sigma^-_1 \sigma^+_2 \sigma^+_3 \sigma^+_4 \bar{\omega}^6 \, e^{(i\frac{1}{2}\bar{\eta}_1)} e^{(-i\frac{1}{2}\bar{\eta}_2)} \, e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})},$$

It is observed that in this toy model $\lambda_b(M_{\text{string}}) = \lambda_\tau(M_{\text{string}})$ and it will be sufficient to calculate one of the two. This $SU(5)$ relation [35] is a reflection of the underlying $SO(10)$ symmetry at the level of the NAHE set. As is evident from Eq. (36), such a residual symmetry does not necessarily survive the $SO(10)$ symmetry breaking vectors beyond the NAHE set. Other superstring standard–like models can therefore yield $\lambda_b(M_{\text{string}}) \neq \lambda_\tau(M_{\text{string}})$.

The picture changed vertex operator for the $\bar{\Phi}^b_{2(-1)}$ field is obtained from Eq. (13),

$$\bar{\Phi}_{2(0)}(z) = \lim_{w \to z} e^c(w) T_F(w) \bar{\Phi}_{2(-1)}(z). \tag{38}$$

Using the OPE

$$e^{i\alpha J}(w) e^{i\beta J}(z) \sim (w-z)^{\alpha\cdot\beta} \, e^{i(\alpha+\beta)J}$$

and Eq. (22a) we obtain

$$\begin{aligned}
\bar{\Phi}^b_{2(0)} = \frac{i}{2} \{ &e^{(i\frac{1}{2}\chi_{12})} e^{(-i\frac{1}{2}\chi_{34})} (y_1\sigma^+_1\sigma^+_2 + i\omega_2\sigma^-_1\sigma^-_2)\sigma^+_3\sigma^+_4 \\
+ &e^{(-i\frac{1}{2}\chi_{12})} e^{(i\frac{1}{2}\chi_{34})} (y_3\sigma^-_3\sigma^+_4 + i\omega_4\sigma^+_3\sigma^-_4)\sigma^-_1\sigma^+_2 \} \\
&\bar{\omega}^6 e^{(i\frac{1}{2}\bar{\eta}_1)} e^{(-i\frac{1}{2}\bar{\eta}_2)} e^{(i\frac{1}{2}KX)} e^{(i\frac{1}{2}K\bar{X})} \, .
\end{aligned} \tag{39}$$

Only the first term contributes to the nonvanishing quartic order correlator, which is given by

$$A_4 = \frac{g^2}{2\pi} \int d^2 z$$

$$\langle \mathrm{e}^{(-\frac{1}{2}c)}(1) \mathrm{e}^{(-\frac{1}{2}c)}(2) \mathrm{e}^{(-c)}(3) \rangle$$

$$\langle \mathrm{e}^{(\mathrm{i}\frac{1}{2}\chi_{12})}(3) \mathrm{e}^{(-\mathrm{i}\frac{1}{2}\chi_{12})}(4) \rangle$$

$$\langle \mathrm{e}^{(\mathrm{i}\frac{1}{2}\chi_{34})}(1) \mathrm{e}^{(\mathrm{i}\frac{1}{2}\chi_{34})}(2) \mathrm{e}^{(-\mathrm{i}\frac{1}{2}\chi_{34})}(3) \mathrm{e}^{(-\mathrm{i}\frac{1}{2}\chi_{34})}(4) \rangle$$

$$\langle \mathrm{e}^{(-\mathrm{i}\frac{1}{2}\zeta_2)}(1) \mathrm{e}^{(-\mathrm{i}\frac{1}{2}\zeta_2)}(2) \mathrm{e}^{(\mathrm{i}\zeta_2)}(4) \rangle$$

$$\langle \mathrm{sigmas} \rangle$$

$$\langle \mathrm{e}^{(\mathrm{i}\frac{1}{2}\bar\zeta_2)}(1) \mathrm{e}^{(-\mathrm{i}\frac{1}{2}\bar\zeta_2)}(2) \rangle$$

$$\langle \mathrm{e}^{(\mathrm{i}\bar J_{3,1}\cdot\bar W_{3,1})}(1) \mathrm{e}^{(\mathrm{i}\bar J_{3,2}\cdot\bar W_{3,2})}(2) \mathrm{e}^{(\mathrm{i}\bar J_{1,2}\cdot\bar W_{1,2})}(3) \rangle$$

$$\langle \mathrm{e}^{(-\mathrm{i}\frac{1}{2}\bar\eta_2)}(1) \mathrm{e}^{(-\mathrm{i}\frac{1}{2}\bar\eta_2)}(2) \mathrm{e}^{(\mathrm{i}\frac{1}{2}\bar\eta_2)}(3) \mathrm{e}^{(\mathrm{i}\frac{1}{2}\bar\eta_2)}(4) \rangle$$

$$\left\langle \prod_{i=1}^{4} \mathrm{e}^{[\mathrm{i}\frac{1}{2}K_i X(i)]} \mathrm{e}^{[\mathrm{i}\frac{1}{2}K_i \bar X(i)]} \right\rangle \tag{40}$$

where

$$\langle \mathrm{sigmas} \rangle = \langle \sigma_1^+(3)\sigma_1^+(4) \rangle$$
$$\langle \sigma_2^+(1)\sigma_2^+(2)\sigma_2^+(3)\sigma_2^+(4) \rangle$$
$$\langle \sigma_3^+(3)\sigma_3^+(4) \rangle$$
$$\langle \sigma_4^+(3)\sigma_4^+(4) \rangle$$
$$\langle \sigma_6^-(1)\sigma_6^+(2)\bar\omega_6(4) \rangle . \tag{41}$$

The correlator is evaluated by using Eqs. (19–23). In addition the correlator due to the kinetic quantum numbers yields,

$$\left\langle \prod_{i=1}^{4} \mathrm{e}^{[\mathrm{i}\frac{1}{2}K_i X(i)]} \mathrm{e}^{[\mathrm{i}\frac{1}{2}K_i \bar X(i)]} \right\rangle = \prod_{i<j} |z_{ij}|^{\frac{1}{2}K(i)\cdot K(j)}$$

$$= |z_\infty|^{-\frac{1}{4}(s+t+u)} |1-z|^{-\frac{u}{4}} |z|^{-\frac{s}{4}} \tag{42}$$

where s, t and u are the usual Mandelstam variables with,

$$s = -2\ K(1)\cdot K(2)\ =\ -2\ K(3)\cdot K(4)$$

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$$t = -2\ K(1) \cdot K(3)\ =\ -2\ K(2) \cdot K(4)$$
$$u = -2\ K(1) \cdot K(4)\ =\ -2\ K(2) \cdot K(3) \tag{43}$$

and $(s+t+u) = 0$. All the Standard Model states from the NS sectors, the sectors b_j (j=1,2,3), and the sector $b_1 + b_2 + \alpha + \beta$ fall into representations of the underlying $SO(10)$ symmetry. We can use the corresponding weights under the $SO(10)$ symmetry to evaluate the correlator under the $SO(10)$ subgroup. Therefore, the correlator under the $SO(10)$ gauge group yields,

$$\langle \mathrm{e}^{(\mathrm{i}\bar{J}_{\bar{3},1}\cdot\bar{W}_{\bar{3},1})}(z_1)\mathrm{e}^{(\mathrm{i}\bar{J}_{\bar{3},2}\cdot\bar{W}_{3,2})}(z_2)\mathrm{e}^{(\mathrm{i}\bar{J}_{1,2}\cdot\bar{W}_{1,2})}(z_3) \rangle =$$
$$\bar{z}_{12}^{W_{16}\cdot W_{16'}}\bar{z}_{13}^{W_{16}\cdot W_{10}}\bar{z}_{23}^{W_{16'}\cdot W_{10}}C_{16\cdot16\cdot10} \tag{44a}$$

where

$$W_{16} \cdot W_{16'} = -3/4 \ ;\ W_{16} \cdot W_{10} = W_{16'} \cdot W_{10} = -1/2 \tag{44b}$$

and

$$W_{16} + W_{16'} + W_{10} = 0 \quad ; \quad W_{16}^2 = W_{16'}^2 = 5/4 \ ;\ W_{10}^2 = 1 \tag{44c}$$

SL(2,C) invariance is used to fixed three of the points, $z_1 = \infty$, $z_2 = 1$, $z_3 = z$ $z_4 = 0$. Using the OPEs and collecting all the terms we obtain the one dimensional complex integral,

$$I = \int d^2z|z|^{-\frac{s}{4}-1}|1-z|^{-\frac{u}{4}-\frac{7}{4}}(1+|z|+|1-z|)^{\frac{1}{2}}. \tag{45}$$

To obtain the contact term we set $s = u = 0$. The integral is then evaluated numerically by shifting $z \to 1 - z$ and using polar coordinates,

$$I = 2 \int_0^\infty dr \int_0^\pi d\theta\ r^{-3/4}(1 - 2r\cos\theta + r^2)^{-1/2}\{1 + r + \sqrt{1 - 2r\cos\theta + r^2}\}^{1/2} \approx 77.7$$

and

$$A_4 = \frac{g^2}{2\pi}\frac{1}{4}I. \tag{46}$$

In general, to determine the contact term at this stage, one needs to subtract the field theory contributions to the four point amplitude [36]. Possible graviton,

gauge and massless matter fields must be accounted for. However, for the terms in Eq. (37), the charges of the fields involved are such that no graviton or gauge boson exchanges are possible. This can be seen from superfield diagrams for the gauge boson : $\Phi^\dagger e^{gV}\Phi$, and the fact that $\Phi^\dagger$ cannot appear in the superpotential together with Φ. In general, gauge boson exchange is only expected in D–terms. Graviton exchange is forbidden because of gauge symmetry, as two of the fields must annihilate into a singlet to allow graviton propagation, which is not the case for the terms in Eq. (37). Thus A_4 is directly related to the coefficient of the nonrenormalizable term in the superpotential.

$$W_4 = \frac{g}{2\pi}\frac{1}{4}\frac{I}{M}(d^c_{L_2}Q_2 h'_{45}\bar\Phi_2 + e^c_{L_2}L_2 h'_{45}\bar\Phi_2) \tag{47}$$

where the relation

$$\frac{1}{2}g\sqrt{2\alpha'} = \frac{\sqrt{8\pi}}{M_{\text{Pl}}} = \frac{1}{2}\frac{1}{M} \tag{48}$$

has been used. These quartic order terms in the superpotential will become effective mass terms for the bottom quark and tau lepton provided that $\bar\Phi_2$ get a VEV of order M.

6. Calculation of the effective Yukawa couplings

The massless spectrum of the free fermionic models contains an "anomalous" $U(1)$ symmetry. The "anomalous" $U(1)$ generates a Fayet-Iliopoulos D–term at the one–loop level in string perturbation theory. The Fayet-Iliopoulos D–term breaks supersymmetry at the Planck scale and destabilizes the string vacuum. The vacuum is stabilized and supersymmetry is restored by giving VEVs to some Standard Model singlets in the massless string spectrum. The allowed VEVs are constrained by requiring that the vacuum is F and D flat. The set of constraints on the allowed VEVs is summarized in the following set of equations:

$$D_A = \sum_k Q^A_k |\chi_k|^2 + \frac{g^2}{192\pi^2}Tr(Q_A) = 0 \tag{49a}$$

$$D_j = \sum_k Q^j_k |\chi_k|^2 = 0 \tag{49b}$$

$$\langle W\rangle = \langle\frac{\partial W}{\partial\eta_i}\rangle = 0 \tag{49c}$$

where χ_k are the fields that get a VEV and $Q_k{}^j$ is their charge under the $U(1)_j$ symmetry. The set $\{\eta_j\}$ is the set of all chiral superfields.

The solution to the set of Eqs.(49a,49b) must be positive definite, since $|\chi_k|^2 \geq 0$. However, as the total charge of these singlets must have $Q_A < 0$ to cancel the "anomalous" $U(1)$ D–term equation, in many models a phenomenologically realistic solution was not found [17,37]. Among the free fermionic standard–like model, that use the NAHE set to obtain three generations, the only models that were found to admit a solution are models with $\Delta_{1,2,3} = 1$. These models therefore have cubic level Yukawa couplings only for $+2/3$ charged quarks. Several examples exist of models with mass terms for both $+2/3$ and $-1/3$ charged quarks and which do not seem to admit a phenomenologically viable solution. This, of course, may be just a reflection of the limited model search that has been performed to date.

The order of magnitude of the VEVs $\langle \chi_j \rangle$ is determined by the Fayet–Iliopoulos term. Because the Fayet–Iliopoulos term is generated at the one–loop level in string perturbation theory, these VEVs can be naturally suppressed relative to the string–related scale, $M \equiv M_{\rm Pl}/2\sqrt{8\pi} \approx 1.2 \cdot 10^{18}$ GeV. The exact suppression factors depend on the details of specific solutions to the set of F and D flatness constraints. Consequently, some of the nonrenormalizable, order–n terms become effective renormalizable terms with effective Yukawa couplings,

$$\lambda = c_n \left(\frac{\langle \phi \rangle}{M} \right)^{n-3}. \tag{50}$$

From Eq. (37) we observe that in order to obtain nonvanishing bottom quark and tau lepton mass terms in this specific toy model, we need to find a solution to the set of D and F constraints with, $\Phi_1 \neq 0$ or $\bar{\Phi}_2 \neq 0$. One explicit solution to the set of constraints is given by the set $\{\Phi_{45}, \bar{\Phi}_{13}, \Phi_{13}, \bar{\Phi}_2\}$, with

$$|\langle \Phi_{45} \rangle|^2 = 3|\langle \bar{\Phi}_2 \rangle|^2 = 3|\langle \Delta_{13} \rangle| = \frac{3g^2}{16\pi^2} \frac{1}{2\alpha'} = \frac{3g^4}{16\pi^2} \frac{1}{M^2} \tag{51}$$

where $\Delta_{13} = (|\bar{\Phi}_{13}|^2 - |\Phi_{13}|^2)$.

With this solution, after inserting the VEV of $\bar{\Phi}_2$ and the coefficients of the quartic order correlators, the effective bottom quark and tau lepton Yukawa couplings are given by,

$$\lambda_b = \lambda_\tau = \frac{I}{32\pi^2} g^3 \approx 0.25 g^3. \tag{52}$$

The top quark mass prediction, Eq. (1), was obtained by taking $g \sim 1/\sqrt{2}$ at the unification scale. The three Yukawa couplings are then run to the low energy

scale by using the MSSM one–loop RGEs. The bottom and top quarks masses are given by

$$m_t(\mu) = \lambda_t(\mu)v_1 = \lambda_t(\mu)\frac{v_0}{\sqrt{2}}\sin\beta \qquad m_b(\mu) = \lambda_b(\mu)v_2 = \lambda_b(\mu)\frac{v_0}{\sqrt{2}}\cos\beta, \quad (53)$$

where $v_0 = 2M_W/g_2 = 246 GeV$ and $\tan\beta = v_1/v_2$. The bottom quark mass, $m_b(M_Z)$ and the W-boson mass, $M_W(M_W)$, are used to calculate the two electroweak VEVs, v_1 and v_2. Using the relation,

$$m_t \approx \lambda_t(M_Z)\sqrt{\frac{2M_W^2}{g_2^2(M_W)} - \left(\frac{m_b(M_Z)}{\lambda_b(m_Z)}\right)^2} \qquad (54)$$

the top quark mass prediction, Eq. (1), is obtained.

The solution, Eq. (51), is of course not unique. It is important to examine what is the range of $\bar{\Phi}_2$ and consequently of $\lambda_b(M_{\text{string}})$ and $\lambda_\tau(M_{\text{string}})$ and how they affect the low energy prediction of the heavy fermion masses. A simple modification of the above solution is obtained by adding the field $\bar{\Phi}_1$ to $\{\Phi_{45}, \bar{\Phi}_2, \Phi_{13}, \bar{\Phi}_{13}\}$ that were used in the above solution. The VEVs of Φ_{45} and Δ_{13} remain the same. We now obtain the equation,

$$|\langle\bar{\Phi}_2\rangle|^2 + |\langle\bar{\Phi}_1\rangle|^2 = \frac{g^2}{16\pi^2}\frac{1}{2\alpha'}, \qquad (55)$$

and,

$$\langle\bar{\Phi}_2\rangle = \sqrt{\frac{g^2}{16\pi^2}\frac{1}{2\alpha'} - |\langle\bar{\Phi}_1\rangle|^2} = \frac{g^2}{4\pi}M\sqrt{1 - \frac{16\pi^2}{g^4M^2}|\langle\bar{\Phi}_1\rangle|^2} \ . \qquad (56)$$

Consequently, with this solution, $\langle\bar{\Phi}_2\rangle$ varies between

$$0 \leq \langle\bar{\Phi}_2\rangle \leq \frac{g^2}{4\pi}M \ , \qquad (57)$$

and the bottom quark and tau lepton Yukawa couplings vary accordingly,

$$0 \leq \lambda_b(M_{\text{string}}) = \lambda_\tau(M_{\text{string}}) \leq \frac{I}{32\ \pi^2}g^3\sqrt{1 - \frac{16\pi^2}{g^4M_2}|\langle\bar{\Phi}_1\rangle|^2} \ . \qquad (58)$$

A lower bound can only be imposed from the physical bottom quark and tau lepton masses.

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7. The effect of intermediate matter thresholds

In the preceding sections we calculated the heavy generation Yukawa couplings at the string scale. The next step in the analysis of the fermion masses is to renormalize the Yukawa couplings from the string scale to the electroweak scale. The spectrum of massless states and the Yukawa couplings are those that appear in the specific superstring derived standard–like toy model. In the proceeding analysis I will make some motivated assumptions with regard to the mass scales of various states that exist in the specific string model which is being analyzed.

Superstring theory in general and free fermionic models in particular predict that all gauge couplings are unified at the string unification scale [38], which is numerically of the order of [39]

$$M_{\text{string}} \approx g_{\text{string}} \times 5 \times 10^{17} \text{GeV} , \tag{59}$$

where g_{string} is the unified gauge coupling. Assuming that the particle content below the string scale consist only of the MSSM particle spectrum, results in disagreement with the values extracted at LEP for $\alpha_{\text{strong}}(M_Z)$ and $\sin^2\theta_W(M_Z)$. In Ref. [23] it was shown, in a wide range of realistic free fermionic models, that heavy string threshold corrections, non-standard hypercharge normalizations [24], light SUSY thresholds or intermediate gauge structure, do not resolve the disagreement with $\alpha_{\text{strong}}(M_Z)$ and $\sin^2\theta_W(M_Z)$. Instead, as was previously suggested [37,22,40], the problem may be resolved in the free fermionic models due to the existence of additional color triplets and electroweak doublets beyond the MSSM. Indeed, additional color triplets and electroweak doublets in vector–like representations, beyond the MSSM, in general appear in the massless spectrum of the realistic string models. The number of such states and their mass scales are highly model dependent. Mass terms for these extra states may arise from cubic or higher–order non–renormalizable terms in the superpotential. In general, the masses of the extra states are suppressed relative to the string scale because of the suppression of the non–renormalizable terms relative to the cubic level terms. Additional mass scales that are suppressed relative to the Planck scales may arise, for example, by condensation in the hidden sector [41]. These additional matter thresholds also affect the evolution of the Yukawa couplings, and it is therefore necessary to include their effect in the analysis of the fermion masses.

In the superstring derived standard–like models such additional color triplets and electroweak doublets are obtained from exotic sectors that arise from the

additional vectors α, β and γ. For example, the model of Ref. [22] is obtained from the model of Ref. [16] by the change of a GSO phase,

$$c\begin{pmatrix} \gamma \\ 1 \end{pmatrix} = -1 \rightarrow c\begin{pmatrix} \gamma \\ 1 \end{pmatrix} = +1 \ . \tag{60}$$

This GSO phase change preserves the spectrum and interactions of the massless states which arise from the basis vectors $\{1, S, b_1, b_2, b_3, \alpha, \beta\}$. The states and charges which are generated by these partial set of basis vectors under the four dimensional gauge are therefore identical to those in the model of Ref. [16]. The effect of the phase change in Eq. (60) is to modify the spectrum from sectors which contain the basis vector $\pm\gamma$. Thus, this phase change does not affect the calculation of the Yukawa couplings in the preceding section. Therefore, the heavy generation Yukawa couplings in this model are still given by Eqs. (34,58). The effect of the GSO phase change, Eq. (60), is to modify the massless states from the sectors $b_1 + b_3 + \alpha \pm \gamma$ and $b_2 + b_3 + \beta \pm \gamma$. This model contains in its spectrum two pairs of $(\overline{3}, 1)_{1/3}$ color triplets from these sectors, with one-loop beta-function coefficients,

$$b_{D_1, \bar{D}_1, D_2, \bar{D}_2} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{5} \end{pmatrix}; \tag{61}$$

one additional pair of color triplets, $(\overline{3}, 1)_{1/6}$, from the sector $1 + \alpha + 2\gamma$ with,

$$b_{D_3, \bar{D}_3} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{20} \end{pmatrix}; \tag{62}$$

and three pairs of $(1, 2)_0$ doublets with

$$b_{\ell, \bar{\ell}} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \end{pmatrix}. \tag{63}$$

The one–loop and two–loop β function coefficients of the states from the sectors b_j, the Neveu–Schwarz sector, and the sector $b_1 + b_2 + \alpha + \beta$ are identical to those

of the MSSM representations. Similarly, for any state with standard $SU(5)$ embedding the β–function coefficients are the same as for the $SU(5)$ representations. The two–loop β function coefficients of the exotic matter are

$$b_{D_3,\bar{D}_3} = \begin{pmatrix} \frac{1}{9} & 0 & \frac{4}{15} \\ 0 & 0 & 0 \\ \frac{1}{30} & 0 & \frac{17}{3} \end{pmatrix} \quad \text{and} \quad b_{\ell,\bar{\ell}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{7}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{64}$$

This particular combination of representations and hypercharge assignments opens up a sizable window in which the low–energy data and string unification can be reconciled. The standard–like models predict $\sin^2\theta_W = 3/8$ at the unification scale due to the embedding of the weak hypercharge in $SO(10)$. The $SO(10)$ embedding of the weak hypercharge in these models enables string scale gauge coupling unification to be in agreement with the low energy data. Of course, there exist a large number of possible scenarios for the mass scales of the extra states and classification of all these possibilities is beyond the scope of this paper. It is found, for example, that if the extra triplets, $\{D_1, \bar{D}_1, D_2, \bar{D}_2, D_3, \bar{D}_3\}$ all have equal masses in the approximate range $2 \times 10^{11} \leq M_3 \leq 7 \times 10^{13}$ GeV with the doublet masses in the corresponding range $9 \times 10^{13} \leq M_2 \leq 7 \times 10^{14}$ GeV, then agreement with LEP data can be obtained [23].

The analysis proceeds as follows. The heavy generation Yukawa couplings, λ_t, λ_b and λ_τ, are renormalized from the string scale to the electroweak scale by running the two–loop supersymmetric RGEs for the gauge and Yukawa couplings, including the contribution of the extra matter. The Yukawa couplings at M_{string} are given by Eqs. (34,58) in terms of g_{string}. The top quark Yukawa coupling is given by Eq. (34). The bottom quark and tau lepton Yukawa couplings as a function of $\langle \bar{\Phi}_1 \rangle$ are given by Eq. (58). The string unification scale, M_{string} is determined by Eq. (59). The unified gauge coupling, α_{string}, is varied in the range $0.03 - 0.07$. The gauge coupling heavy string threshold corrections in this toy model were analyzed in Ref. [23]. The two–loop coupled supersymmetric RGEs for the gauge and Yukawa couplings are then evolved to the extra doublets and triplets thresholds. The three color triplet pairs and three electroweak doublet pairs, beyond the MSSM, are assumed to be degenerate at the mass scales M_3 and M_2 respectively. The extra doublet and triplet thresholds are varied in the ranges

$$1 \times 10^{13} \leq M_2 \leq 1 \times 10^{16} \text{ GeV}$$

$$9 \times 10^9 \leq M_3 \leq 1 \times 10^{12} \text{ GeV}, \tag{65}$$

respectively. The contribution of each threshold to the β–function coefficients is removed in a step approximation. The coupled two–loop RGEs are then evolved to the approximate top quark mass scale, $m_t \approx 175$ GeV. At this scale the top quark Yukawa coupling and $\alpha_{\text{strong}}(m_t)$ are extracted, and the contribution of the top quark to the RGEs is removed. The two–loop supersymmetric RGEs are then evolved to the Z mass scale and agreement with the experimental values of $\alpha_{\text{strong}}(M_Z)$, $\sin^2 \theta_W(M_Z)$, and $\alpha_{\text{em}}^{-1}(M_Z)$ is imposed. In this section all the superpartners are assumed to be degenerate at M_Z. Possible corrections due to the superparticle spectrum will be examined in the next section. The gauge couplings of $SU(3)_{\text{color}} \times U(1)_{\text{em}}$ are then extrapolated from the Z–boson mass scale to the bottom quark mass scale. The running bottom quark and tau lepton masses are evolved back from their physical mass scale to the Z mass scale by using the three–loop QCD and two–loop QED RGEs [42]. The bottom mass is then used to extract the running top quark mass, using Eq. (54). The physical top quark mass is given by,

$$m_t(physical) = m_t(m_t)(1 + \frac{4}{3\pi}\alpha_{\text{strong}}(m_t)), \tag{66}$$

where $m_t(m_t)$ is given by Eq. (54).

From Eq. (37) we observe that in this model $\lambda_b = \lambda_\tau$ at the string unification scale. Consequently, an additional prediction for the mass ratio

$$\lambda_b(M_Z)/\lambda_\tau(M_Z) = m_b(M_Z)/m_\tau(M_Z) \tag{67}$$

is obtained. λ_b and λ_τ are extrapolated from the string unification scale to the Z–mass scale using the two–loop RGEs with the intermediate matter thresholds, as described above. The gauge couplings of $SU(3)_{\text{color}} \times U(1)_{\text{em}}$ are then extrapolated to the bottom quark mass scale. The bottom quark and tau lepton masses are then extrapolated to the Z mass scale. We can then compare the predicted ratio on the left–hand side of Eq. (67) with the experimentally extrapolated ratio on the right–hand side.

Low–Energy Experimental Inputs

For the subsequent analysis, the input parameters are the tree-level string prediction, Eq. (59), and g_{string} is varied as described above. The mass of the Z–boson is [43]

$$M_Z \equiv 91.161 \pm 0.031 \text{ GeV} . \tag{68}$$

The RGE's are run from the string scale to the Z scale. At the Z scale we obtain predictions for the gauge parameters α_{strong}, $\sin^2 \theta_W$ and α_{em}. These predictions are constrained to be in the experimentally allowed regions [43],

$$\alpha_{\text{strong}}(M_Z) = 0.12 \pm 0.01,$$
$$\sin^2 \theta_W(M_Z) = 0.232 \pm 0.001$$
$$\alpha_{\text{em}}^{-1}(M_Z) = 127.9 \pm 0.1 . \tag{69}$$

Note that these values are obtained in the $\overline{MS}$–renormalization scheme while the predictions from the supersymmetric RGE's are obtained in the $\overline{DR}$–renormalization scheme. The predictions are converted to the $\overline{MS}$–renormalization scheme by using the conversion factors,

$$\frac{1}{\alpha_i^{\overline{DR}}} = \frac{1}{\alpha_i^{\overline{MS}}} - \frac{C_{A_i}}{12\pi} \tag{70}$$

where the C_{A_i} are the quadratic Casimir coefficients of the adjoint representations of the gauge factors: $C_{A_3} = 3$, $C_{A_2} = 2$, $C_{A_1} = 0$.

The running bottom quark and tau lepton masses in the $\overline{MS}$–renormalization scheme are [43]

$$m_b(m_b) = 4.4 \pm 0.3 \quad \text{GeV} \qquad \text{and}$$
$$m_\tau(m_\tau) = 1777.1^{+0.4}_{-0.5} \text{ MeV} . \tag{71}$$

These values are extrapolated from the low energy regime to the Z mass scale using the three–loop QED and two–loop QCD RGEs. The conversion from $\overline{MS}$ to $\overline{DR}$ increases m_b by roughly half a percent and has virtually no effect on m_τ [45].

Numerical results

In this section all the superpartners are assumed to be degenerate at the Z mass scale. Some possible corrections due to the splitting in the supersymmetric mass spectrum are examined in the next section. It should be emphasized that the numerical analysis is not intended as a complete analysis of the parameter space. The purpose of the numerical analysis is to illustrate how stringy calculations may be confronted with experimental data and what are the still missing pieces in trying to improve the predictability of the string derived models.

The heavy generation Yukawa couplings are given by Eqs. (34,58). In Fig. (1) the dependence of $\lambda_b(M_{\text{string}}) = \lambda_\tau(M_{\text{string}})$ on $\langle \bar{\Phi}_1 \rangle$ is shown. The lower limit arises from requiring that $\text{Im}(\lambda_t(M_Z)) = 0$. In all of the figures, agreement with the gauge parameters at the Z–boson mass scale is imposed. In Fig. (2), the predicted physical top quark mass $m_t(m_t)$ is plotted versus the predicted value of $\tan\beta$. From the figure we observe that the predicted top quark mass varies in the interval $90\ \text{GeV} \leq m_t \leq 205\ \text{GeV}$. In Fig. (3) the top quark mass is plotted versus $\lambda_t(m_t)$. Although the top Yukawa coupling is near its fixed point, the predicted top quark mass varies over a wide range. This is of course expected as it merely reflects the dependence of the calculated top quark mass on the bottom quark Yukawa coupling, which is illustrated in Fig. (4). The dependence on $\lambda_b(M_Z)$ in turn is a result of the freedom in the determination of the bottom Yukawa coupling at the sting scale. In Fig. (5) m_t is plotted versus $\langle \bar{\Phi}_2 \rangle / M$ which demonstrated the dependence of the predicted top quark mass on the nonvanishing VEVs in the DSW mechanism. Similarly, the predicted value of $\tan\beta$ depends on the initial boundary conditions and on $\lambda_b(M_Z)$, which is shown in fig (6). Fig (7) shows the dependence of m_t on $\alpha_{\text{strong}}(M_Z)$. Again although there is a slight increase in the predicted values of m_t as $\alpha_{\text{strong}}(M_Z)$ increases, a strong dependence is not observed.

In Fig. (8) the predicted ratio $\lambda_b(M_Z)/\lambda_\tau(M_Z)$ is plotted versus the experimentally observed ratio $m_b(M_Z)/m_\tau(M_Z)$. The bottom quark mass is varied in the interval $4.1 - 4.7\ \text{GeV}$. It is seen that qualitatively there is very good agreement between the predicted ratio and the experimentally observed ratio. Over some of the parameter space the predicted ratio is somewhat larger than the experimentally observed mass ratio, and better agreement is obtained for the larger values of $m_b \approx 4.5 - 4.7\ \text{GeV}$. It is very important to note that the additional intermediate matter thresholds that are needed to obtain agreement with the gauge parameters are also crucial to maintain the agreement with the b/τ mass ratio. This is due to the dependence of the running bottom mass on the strong coupling. As the intermediate matter states prevent the strong coupling from growing outside

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its experimental bound, they prevent the bottom quark mass from becoming too large. Thus, the intermediate matter thresholds that are required for string gauge coupling unification to be in agreement with the low energy data are also required for obtaining the correct mass ratio m_b/m_τ. Of course, the splitting in the supersymmetric mass spectrum can modify this picture. This will be investigated in the next section. In Fig. (9) m_{top} is plotted versus the ratio $\lambda_b(M_Z)/\lambda_\tau(M_Z)$. It is observed that for the explored region of the parameter space there is no strong dependence of the predicted top quark mass on the Yukawa ratio. This again reflects the fact that the top quark mass mainly depends on the bottom Yukawa, or alternatively on $\tan\beta$. It is noted that the intermediate matter thresholds which are required to maintain the low values of α_{strong}, therefore also prevent the bottom Yukawa from growing too large and consequently maintain the experimentally viable ratio of λ_b/λ_τ.

8. SUSY breaking effects

In the analysis in the previous section it was assumed that all the superpartners are degenerate at the Z–boson mass scale. However, in general the supersymmetric spectrum is split and may induce substantial threshold corrections to the calculation of the fermion masses. Furthermore, In the previous analysis the bottom quark and W–boson masses were used to fix the two electroweak VEVs v_1 and v_2. The fermion masses are then calculated in terms of their Yukawa couplings to the Higgs doublets and in terms of the VEVs of the neutral component of the electroweak doublets. However, in local supersymmetric theories, for given boundary conditions at the unification scale , the electroweak VEVs can be determined from the running of the Renormalization Group Equation and minimization of the one–loop effective potential. In this section, I briefly examine the minimization of the Higgs potential and the effects of the heavy superpartner thresholds. In principle it may be possible to extract the soft SUSY breaking parameters from the superstring models. It should be emphasized however that in this paper the derivation of the SUSY breaking parameters from the superstring models is not attempted. The purpose of this section is to briefly examine the potential effects of the SUSY breaking sector on the fermion mass predictions. An attempt to extract further information on the SUSY breaking sector from the string models will be reported in future work.

The analysis proceeds as follows. The string unification scale is given by Eq. (59), and I take $g_{\text{string}} = 0.824$. The spectrum at the string unification scale consist of the MSSM states plus the additional color triplets and electroweak doublets.

The Renormalization Group Equations are those of the MSSM including the contribution of the extra matter states. The parameters of the SUSY breaking sector consist of the universal trilinear coefficient A_0, and the universal scalar and gaugino masses, m_0 and $m_{1/2}$, respectively. The boundary conditions for the soft SUSY breaking terms at the unification scale are taken to be universal and are varied over a sample of the parameter space (see table 3).

Parameter X	X_i	X_f	ΔX
A_0 (GeV)	-200	200	100
m_0 (GeV)	0	200	100
$m_{1/2}$ (GeV)	100	300	100

Table 3 The range and sampling size of the parameter space. Each free parameter X is sampled in the interval (X_i, X_f) with spacing ΔX between consecutive points.

The heavy generation Yukawa couplings at the unification scale are given by Eqs. (34,58). Similar to the procedure described in the previous section, the RGEs are evolved from the string unification scale to the electroweak scale. The contribution of the extra color triplets and electroweak doublets to the β–function coefficients is removed in a step approximation. As a specific example, the three additional color triplet pairs are taken to be degenerate at $M_3 = 2.8 \times 10^{11}$ GeV with the three pairs of electroweak doublets degenerate at $M_2 = 4.0 \times 10^{13}$ GeV. Agreement with the low energy observables $\alpha_{\rm em}(M_Z)$, $\sin\theta_W(M_Z)$ and $\alpha_s(M_Z)$ is then obtained.

The analysis of electroweak symmetry breaking and the minimization of the Higgs scalar potential is standard and has been examined extensively in the context of the MSSM [46]. Below the intermediate scales of the additional vector–like states the matter spectrum is that of the MSSM. The Higgs part of the MSSM scalar potential is given by,

$$V(H_1, H_2) = (m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2 + B\mu(H_1 H_2 + h.c.) +$$
$$\frac{1}{8}g_2^2(H_1^\dagger \sigma H_1 + H_2^\dagger \sigma H_2)^2 + \frac{1}{8}g'^2(|H_2|^2 - |H_2|^2)^2 + \Delta V, \qquad (72)$$

where $H_1 \equiv \binom{H_1^0}{H_1^-}$ and $H_2 \equiv \binom{H_2^+}{H_2^0}$ are the two complex Higgs fields, $m_{H_1}^2$, $m_{H_2}^2$

and $B\,(\mu)$ are the soft supersymmetric mass parameters renormalized down to the weak scale, $m_3 \equiv B\mu < 0$ and g_2 and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings respectively. The one–loop correction ΔV is given by

$$\Delta V = \frac{1}{64\pi^2} \mathrm{STr}\,\mathcal{M}^4 \left(\ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right), \tag{73}$$

where $\mathrm{STr}\,f(\mathcal{M}^2) = \sum_j (-1)^{2j}(2j+1)\mathrm{Tr}\,f(\mathcal{M}_j^2)$ and $\mathcal{M}_j^2$ are the field–dependent spin-j mass matrices. The one–loop corrections, ΔV, receive contributions from all particle species. It is sufficient however to include only the corrections due to the heavy particles, i.e. the heavy top quark and the heavy superpartners. Requiring a negative eigenvalue to the neutral Higgs mass squared matrix and that the Higgs potential is bounded from below imposes two conditions on the running of the mass parameters: The well known tree level minimization constraints

1. $m_1^2 m_2^2 - m_3^4 < 0$
2. $m_1^2 + m_2^2 - 2|m_3^2| > 0,$

where $m_{1,2}^2 = m_{h_{1,2}}^2 + \mu^2$ and $m_3^2 = B\mu$. Similarly, the constraint that the vacuum does not break color and electric charges is imposed [47].

Ordinarily, at this stage, to minimize the Higgs potential one would fix the Higgs mixing parameter μ and the bilinear coupling B from their initial values at the unification scale and their scaling to the weak scale. The electroweak VEVs, v_1 and v_2, can then be obtained from minimization of the one–loop effective potential. Here however a different procedure is followed. The VEVs v_1 and v_2 are fixed by using the physical bottom quark and W–boson masses at the Z scale. The one–loop potential is then minimized for the μ and B parameters. An initial guess for the minimization values of μ and B is obtained from the two minimization conditions of the tree level superpotential, Eq. (72). The sparticle spectrum is obtained by using the regular parameterization (see for example Ref. [48] for the notation used in this paper). In the analysis of the sparticle spectrum the Yukawa couplings of the two light generations are neglected. The heavy generation mass eigenstates are obtained by diagonalizing the respective 2×2 mass matrices. Similarly the neutralino, chargino and Higgs mass eigenstates are obtained by diagonalizing the respective 4×4 and 2×2 mass matrices. The numerical contributions to the tree–level and one–loop Higgs potential are obtained from Eqs. (72,73). The one–loop effective Higgs potential is then minimized numerically by varying the μ and B

parameters. In this procedure μ and B become the computed parameters at the weak scale. This is possible because the running of the SUSY parameters does not depend on the values μ and B (except for μ itself). The supersymmetric mass spectrum is then recomputed using the minimizing values of μ and B.

The above analysis demonstrates that in principle the predicted values of the top quark mass and the mass ratio $m_b(M_Z)/m_\tau(M_Z)$ can be compatible with the minimization of the one–loop effective Higgs potential. Whether this is indeed realized in the string models awaits further analysis of the SUSY breaking sectors in these models and of the μ parameter. The μ problem is one of the more challenging problems facing supersymmetry and superstring phenomenology. Naive solutions to this problem can be contemplated both in field theoretic supersymmetry and in superstring theory [49]. For example, in the superstring derived standard–like models a possible solution was proposed in which the μ term is generated from a nonrenormalizable term in the superpotential [34]. The nonrenormalizable term contains VEVs that break the $U(1)_{Z'}$, of the $SO(10)$ group, which is orthogonal to the Standard Model weak hypercharge. It is argued that if the $U(1)_{Z'}$ symmetry is broken at an intermediate energy scale then a μ parameter of the required order of magnitude can be generated. Additional symmetries that arise from the compactified degrees of freedom prevent any other μ term from being generated. However, in general, in a realistic solution of the string vacuum most of these additional symmetries need to be broken in order to generate potentially realistic fermion mass matrices [50]. Thus, in a generic realistic solution there is the danger that a μ term of the order of $(\langle\Phi\rangle^{n-1}/M^n)$ will be generated from an order n nonrenormalizable term. It is not unconceivable however that in some string models a residual discrete symmetry or a remnant of a custodial symmetry [28] will remain unbroken and will prevent a large μ term from being generated. Such symmetries in a specific string model have been shown for example to prevent proton decay from dimension four operators to all orders of nonrenormalizable terms. However, whether such a scenario can be realized, is not only model dependent but also depends on the details of the vacuum shift in the application of the DSW vacuum shift.

The splitting in the supersymmetric mass spectrum also induces corrections to the tree level mass predictions. The supersymmetric thresholds affects both the gauge coupling and the Yukawa couplings. The threshold corrections for the gauge couplings and Yukawa couplings received considerable attention in the context of the MSSM [51]. It is not the purpose here to present a detailed numerical investigation of the supersymmetric threshold effects and in general the corrections

are expected to be small. I examine the correction of the bottom quark mass due to the gluino and Higgsino. These corrections have been shown to be important in the case of large $\tan\beta$ [45,52]. The corrected bottom quark mass is given by

$$m_b = \lambda_b v_1 (1 + \Delta(m_b)) \tag{74}$$

where $\Delta(m_b)$ receives contributions coming from bottom squark–gluino loops and top squark–chargino loops, and is given by

$$
\begin{aligned}
\Delta(m_b) = {} & \frac{2\alpha_3}{3\pi} m_{\tilde{g}} \mu \tan\beta I(m_{\tilde{b},+}^2, m_{\tilde{b},-}^2, m_{\tilde{g}}^2) \\
& + \frac{\lambda_t}{4\pi} A_t \mu \tan\beta I(m_{\tilde{t},+}^2, m_{\tilde{t},-}^2, \mu^2),
\end{aligned}
\tag{75}
$$

where $m_{\tilde{g}}$ is the gluino mass, $m_{\tilde{b},\pm}$ and $m_{\tilde{t},\pm}$ are the sbottom and stop mass eigenstates respectively, and A_t is the . The integral function is given by,

$$I(a,b,c) = \frac{ab\ln(a/b) + bc\ln(b/c) + ac\ln(c/a)}{(a-b)(b-c)(a-c)} . \tag{76}$$

In Fig. (10) the corrected bottom quark–tau lepton mass ratio is plotted versus the experimentally extrapolated mass ratio $m_b(M_Z)/m_\tau(M_Z)$ for the sample of points in the parameter space which are given in table 3. In Fig. (11) the predicted mass ratio at the Z scale, divided by the experimentally extrapolated mass ratio, is plotted versus the experimentally extrapolated mass ratio. The bottom quark mass is varied in the range $4.1 - 4.7$ GeV. The predicted mass ratio is seen to vary between $\approx 1 - 1.2$. Thus, the predicted mass ratio is in reasonable agreement with the experimental mass ratio and better agreement is obtained for the larger values of $m_b(m_b)$.

9. Discussion and conclusions

The nature of the electroweak symmetry breaking mechanism and the origin of the fermion masses is one of the important pieces in the puzzle of elementary particle physics. The calculation of the fermion masses from fundamental principles is therefore an important task. Within the context of theories of unification, the fermion masses are expected to arise due to some underlying Planck scale physics. Superstring theory provides at present the best tool to probe Planck scale physics.

The superstring derived standard–like models in the free fermionic formulation possess many attractive properties. An important property of the these models is the possible solution to the problem of proton stability [53]. A second important property of free fermionic models is the prediction $\sin^2\theta_W = 3/8$ at the unification scale due to the embedding of the weak hypercharge in $SO(10)$. This rather common result from the point of view of regular GUTs is highly nontrivial from the point of view of string models. It is only due to the standard $SO(10)$ embedding of the weak hypercharge that free fermionic models can be in agreement with the low energy data. Recently, it was suggested that $\sin^2\theta_W(M_U) = 3/8$ is the preferred value also from considerations of the fermion mass spectrum [54].

Another important property of the superstring derived standard–like models is the existence of three and only three chiral generation in the massless spectrum. This property enhances the predictability of the superstring standard–like models. As a result it is possible to identify the states in the string models with the physical mass eigenstates of the Standard Model.

In this paper I discussed in detail the calculation of the heavy generation masses in the superstring derived standard–like models. In these models the top quark gets a cubic level mass term while the mass terms for the lighter quarks and leptons are obtained from nonrenormalizable terms. The top quark Yukawa coupling and the quartic order correlator of the bottom quark and tau lepton mass terms were calculated in a specific model. The numerical coefficient of the quartic order correlator was calculated explicitly and was shown to be nonzero. The quartic order mass terms produce effective Yukawa couplings for the bottom quark and tau lepton after application of the DSW mechanism. The dependence of the effective Yukawa couplings on the VEV in the DSW mechanism and the implication on the top quark mass prediction was studied in detail. The string–scale coupling unification requires the existence of intermediate matter thresholds. The gauge and Yukawa couplings were run from the string unification scale to the low energy scale in the presence of the intermediate matter. It was shown that LEP precision data for α_{strong}, $\sin^2\theta_W$ and α_{em} as well as the CDF/D0 top quark observation and the b/τ mass relation can all simultaneously be compatible with the superstring derived standard–like models.

Although the calculations were presented in a specific toy model, the features of this toy model that are relevant for the analysis are shared by a large class of superstring standard–like models in the free fermionic formulation. Thus, the results are expected to hold in the larger class of models. Similar analysis can of course also

be carried out in other semi–realistic string models. The above results motivate further analysis of the fermion masses in the superstring derived standard–like models. Several directions should be pursued. The first is to try to extend the analysis to the lighter generations. Potential charm quark mass terms appear at the quintic order of the superpotential. For example, $u_2 Q_2 \bar{h}'_{45} \bar{\Phi}_{23} \Phi_{45}$, can provide a quintic order charm quark mass term. The analysis for such a term involves higher order Ising model correlators and a two dimensional complex integration. Another important direction is the analysis of the supersymmetry breaking sector in the standard–like models and possible corrections due to the dilaton and moduli dependence of the Yukawa couplings [55]. Additional corrections may arise from the infinite tower of heavy string modes [56]. Thus, although much more work is needed to understand how the specific string parameters are fixed by the string physics, we have made the initial steps toward the quantitative confrontation of string models with experimental data.

Acknowledgments

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	ψ^μ	$\{\chi^{12}; \chi^{34}; \chi^{56}\}$	$\bar\psi^1, \bar\psi^2, \bar\psi^3, \bar\psi^4, \bar\psi^5, \bar\eta^1, \bar\eta^2, \bar\eta^3$	$\bar\phi^1, \bar\phi^2, \bar\phi^3, \bar\phi^4, \bar\phi^5, \bar\phi^6, \bar\phi^7, \bar\phi^8$
α	0	$\{0,\,0,\,0\}$	1, 1, 1, 0, 0, 0, 0, 0	1, 1, 1, 1, 0, 0, 0, 0
β	0	$\{0,\,0,\,0\}$	1, 1, 1, 0, 0, 0, 0, 0	1, 1, 1, 1, 0, 0, 0, 0
γ	0	$\{0,\,0,\,0\}$	$\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2}$	$\frac{1}{2},\ 0,\ 1,\ 1,\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ 0$

	$y^3y^6,\ y^4\bar y^4,\ y^5\bar y^5,\ \bar y^3\bar y^6$	$y^1\omega^6,\ y^2\bar y^2,\ \omega^5\bar\omega^5,\ \bar y^1\bar\omega^6$	$\omega^1\omega^3,\ \omega^2\bar\omega^2,\ \omega^4\bar\omega^4,\ \bar\omega^1\bar\omega^3$
α	1, 1, 1, 0	1, 1, 1, 0	1, 1, 1, 0
β	0, 1, 0, 1	0, 1, 0, 1	1, 0, 0, 0
γ	0, 0, 1, 1	1, 0, 0, 0	0, 1, 0, 1

Table 1. A three generations $SU(3) \times SU(2) \times U(1)^2$ model. The choice of generalized GSO coefficients is:

$$c\begin{pmatrix} b_j \\ \alpha, \beta, \gamma \end{pmatrix} = -c\begin{pmatrix} \alpha \\ 1 \end{pmatrix} = -c\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = -c\begin{pmatrix} \beta \\ 1 \end{pmatrix} = c\begin{pmatrix} \gamma \\ 1 \end{pmatrix} = -c\begin{pmatrix} \gamma \\ \alpha, \beta \end{pmatrix} = -1$$

(j=1,2,3), with the others specified by modular invariance and space–time supersymmetry. $\Delta_{1,2,3} = 1 \Rightarrow$ cubic level Yukawa couplings are obtained only for $+\frac{2}{3}$ charged quarks.

	ψ^μ	$\{\chi^{12}; \chi^{34}; \chi^{56}\}$	$\bar\psi^1, \bar\psi^2, \bar\psi^3, \bar\psi^4, \bar\psi^5, \bar\eta^1, \bar\eta^2, \bar\eta^3$	$\bar\phi^1, \bar\phi^2, \bar\phi^3, \bar\phi^4, \bar\phi^5, \bar\phi^6, \bar\phi^7, \bar\phi^8$
α	1	$\{1, 0, 0\}$	1, 1, 1, 1, 1, 1, 0, 0	0, 0, 0, 0, 0, 0, 0, 0
β	1	$\{0, 0, 1\}$	1, 1, 1, 0, 0, 1, 1, 0	1, 1, 1, 1, 0, 0, 0, 0
γ	1	$\{0, 1, 0\}$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, 0, 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0$

	$y^3 y^6, y^4 \bar y^4, y^5 \bar y^5, \bar y^3 \bar y^6$	$y^1 \omega^6, y^2 \bar y^2, \omega^5 \bar\omega^5, \bar y^1 \bar\omega^6$	$\omega^1 \omega^3, \omega^2 \bar\omega^2, \omega^4 \bar\omega^4, \bar\omega^1 \bar\omega^3$
α	1, 0, 0, 1	0, 0, 1, 0	0, 0, 1, 0
β	0, 0, 0, 1	0, 1, 0, 1	1, 0, 1, 0
γ	0, 0, 1, 1	1, 0, 0, 1	0, 1, 0, 0

Table 2. A three generations $SU(3) \times SU(2) \times U(1)^2$ model. The choice of generalized GSO coefficients is:

$$c\begin{pmatrix} \alpha \\ b_j, \beta \end{pmatrix} = -c\begin{pmatrix} \alpha \\ 1 \end{pmatrix} = c\begin{pmatrix} \beta \\ 1 \end{pmatrix} = c\begin{pmatrix} \beta \\ b_j \end{pmatrix} = -c\begin{pmatrix} \beta \\ \gamma \end{pmatrix} = c\begin{pmatrix} \gamma \\ b_2 \end{pmatrix} = -c\begin{pmatrix} \gamma \\ b_1, b_3, \alpha, \gamma \end{pmatrix} = -1$$

(j=1,2,3), with the others specified by modular invariance and space–time supersymmetry. Trilevel Yukawa couplings are obtained for $+2/3$ charged quarks as well as $-1/3$ charged quarks and for charged leptons. $\Delta_1 = 1 \Rightarrow$ Yukawa coupling for $+2/3$ charged quark from the sector b_1. $\Delta_{2,3} = 0 \Rightarrow$ Yukawa couplings for $-1/3$ charged quarks and charged leptons from the sectors b_2 and b_3.
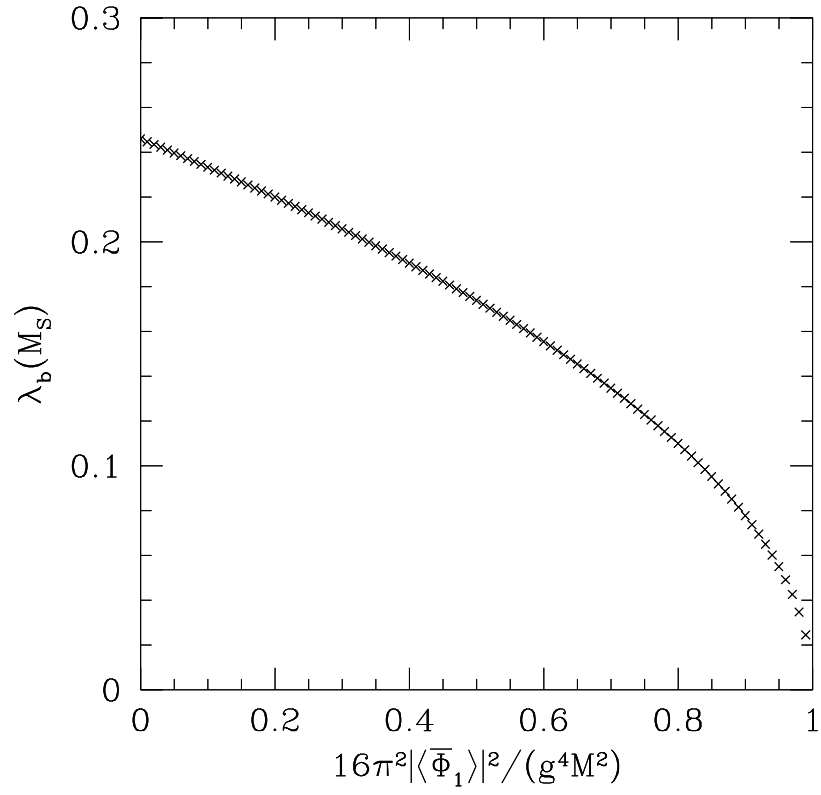
Figure 1. The bottom quark Yukawa coupling at the string unification scale as a function of the VEV $\langle \Phi_1 \rangle$.
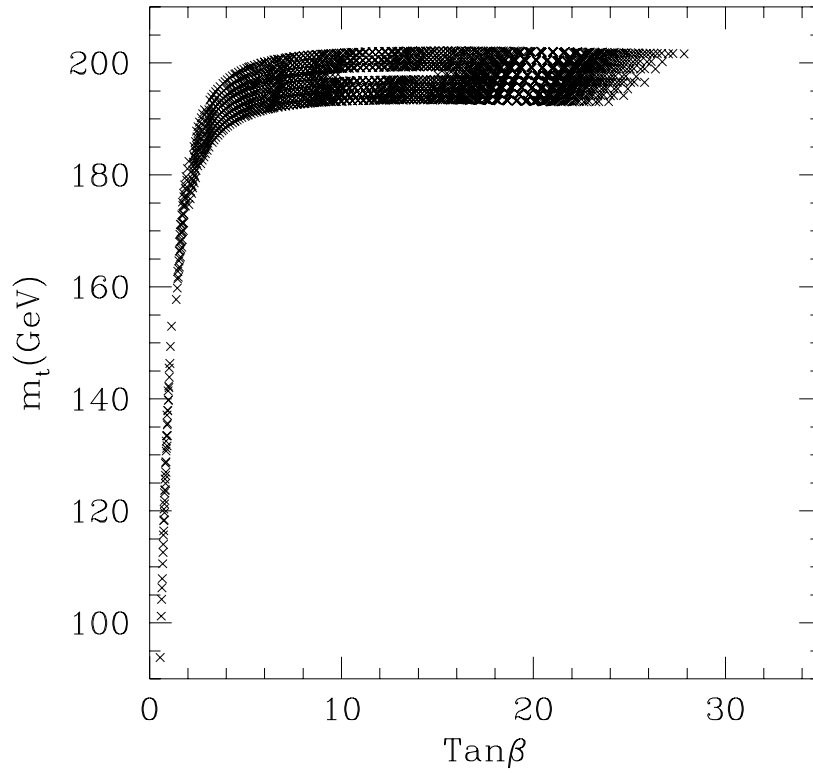
Figure 2. A scatter plot of the physical top quark mass, $m_t(m_t)$ versus the electroweak VEVs ratio, $\tan\beta = v_1/v_2$. Each point in this plot represents a specific choice for the initial boundary conditions for the gauge and Yukawa couplings at the string unification scale and for the mass scales of the intermediate matter states.
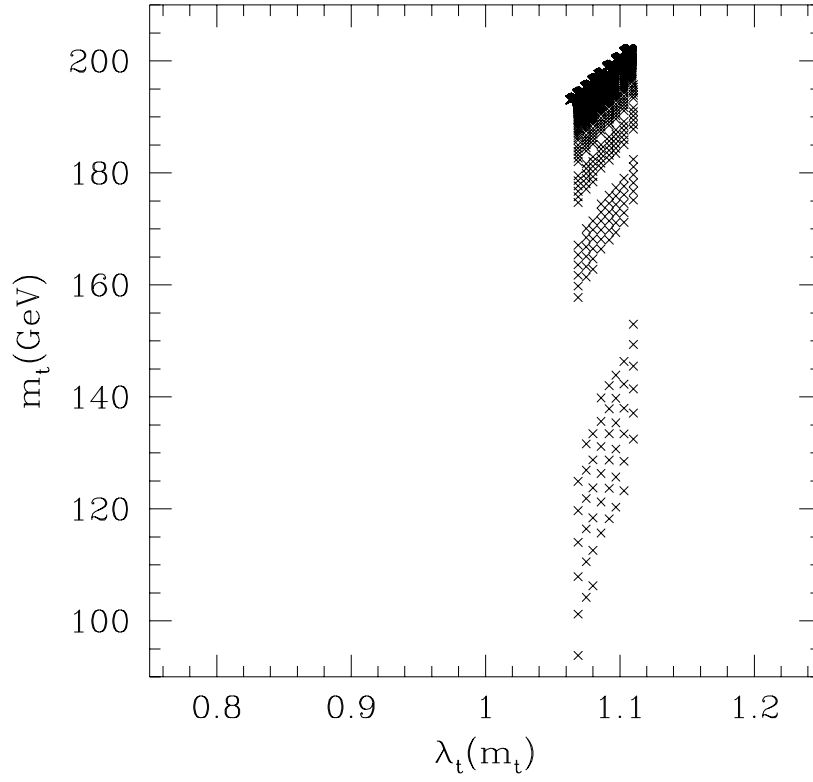
Figure 3. A scatter plot of the physical top quark mass, $m_t(m_t)$ versus the top quark Yukawa coupling $\lambda_t(m_t)$. Each point represents a choice of parameters as in figure 2. $\lambda_t(m_t)$ is found near its fixed point. However, there is a wide variation in $m_t(m_t)$. This reflects the dependence of the predicted top quark mass on the bottom quark Yukawa coupling, or alternatively on the VEVs ratio $\tan\beta$.
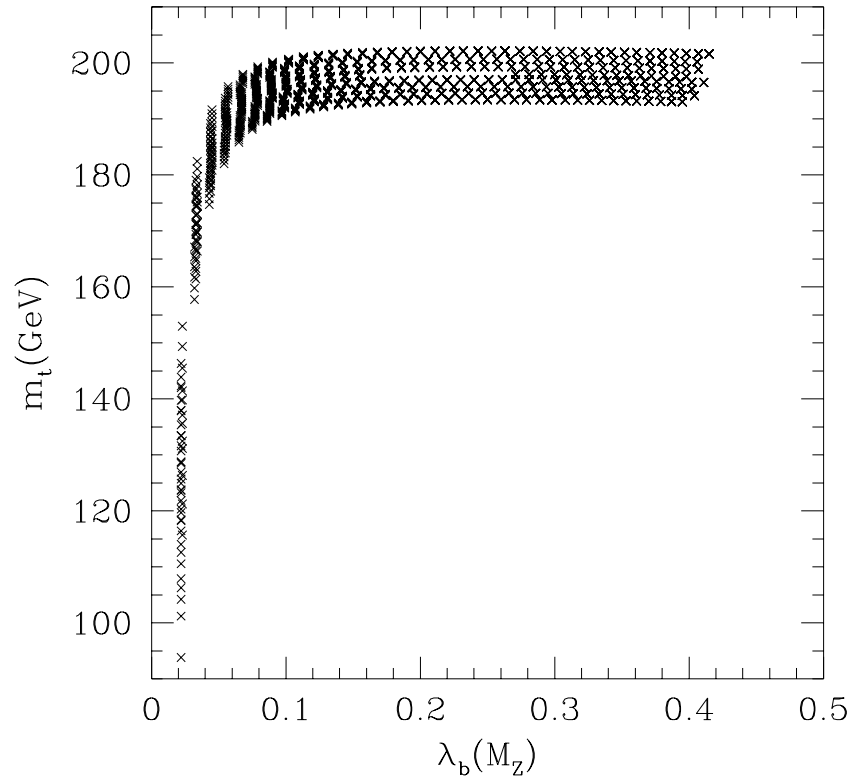
Figure 4. A scatter plot of the physical top quark mass, $m_t(m_t)$ versus the bottom quark Yukawa coupling $\lambda_b(M_Z)$. Each point represents a choice of parameters as in figure 2.
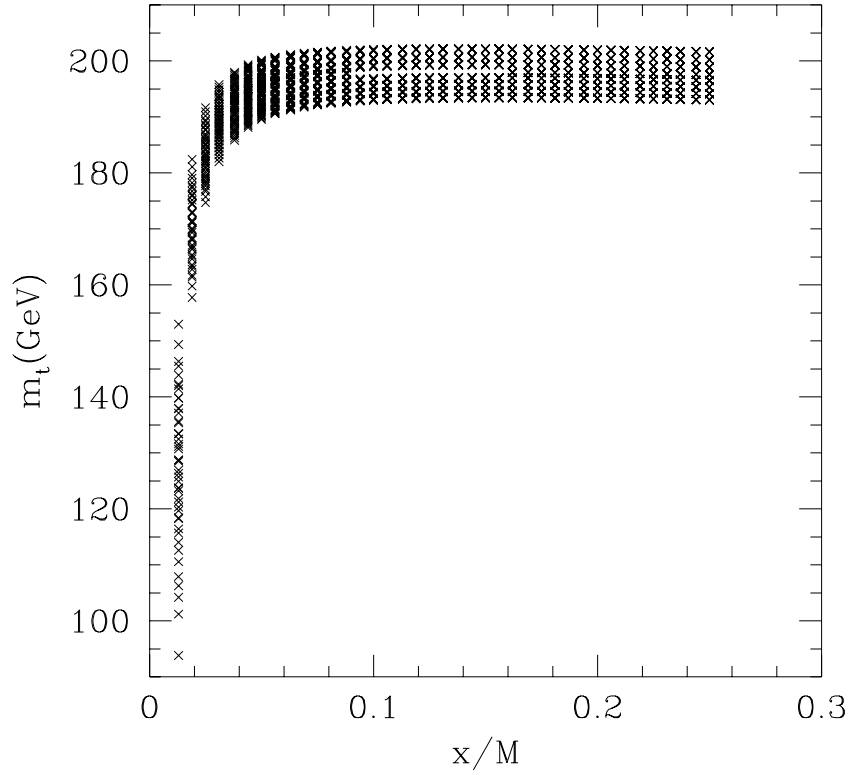
Figure 5. A scatter plot of the physical top quark mass, $m_t(m_t)$ versus the VEV in the DSW mechanism, $\langle \bar{\Phi}_2 \rangle$, which fixes the effective bottom quark and tau lepton Yukawa couplings at the unification scale. Each point represents a choice of parameters as in figure 2.
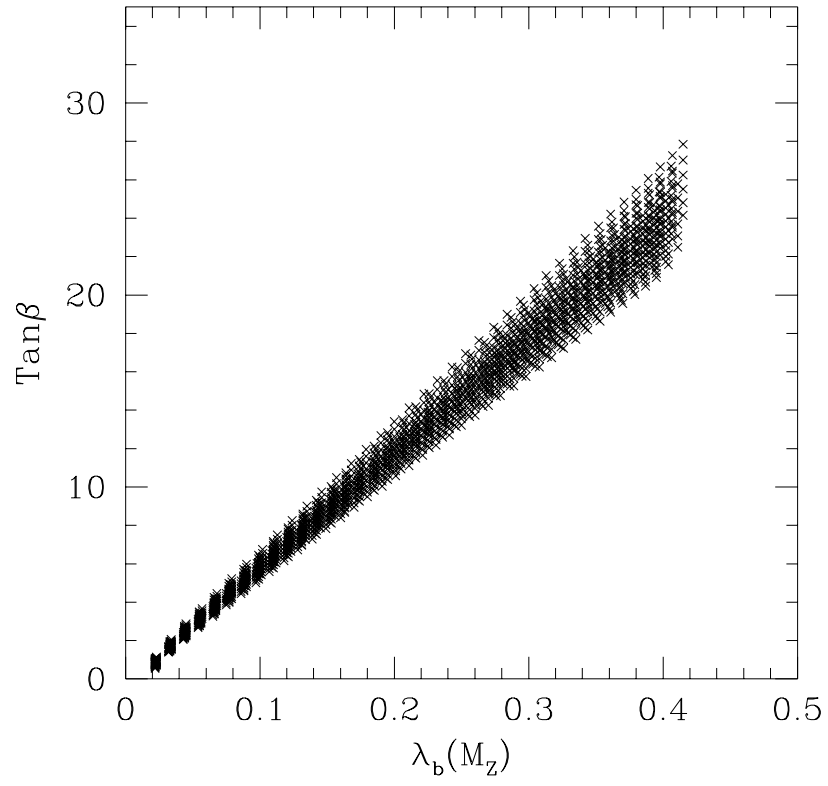
Figure 6. A scatter plot of $\tan\beta$ versus $\lambda_b(M_Z)$. Each point represents a choice of parameters as in figure 2.
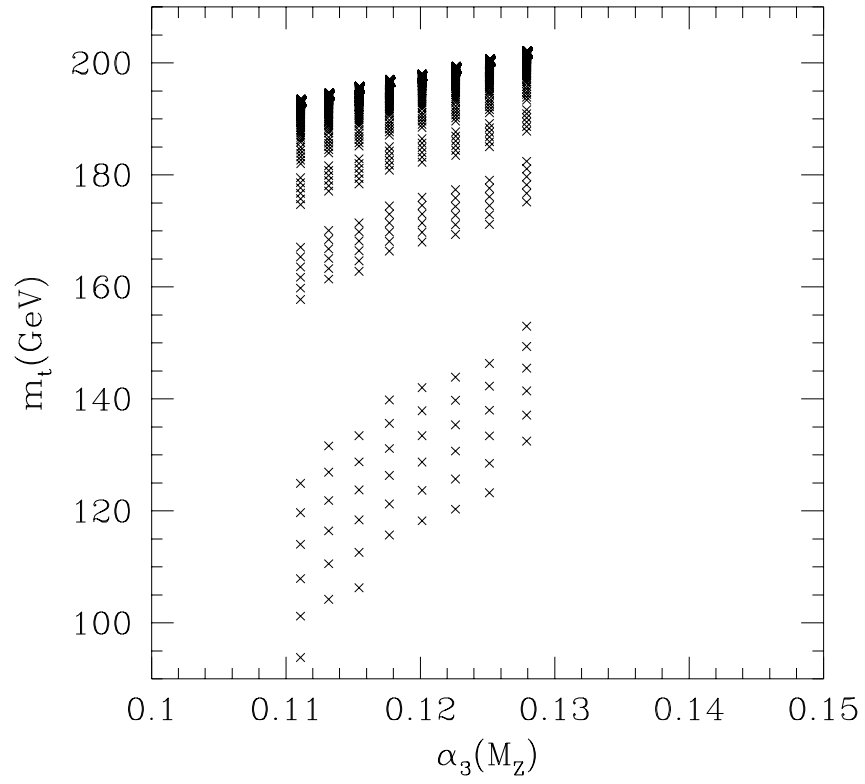
Figure 7. A scatter plot of the physical top quark mass, $m_t(m_t)$ versus the strong coupling $\alpha_{\mathrm{strong}}(M_Z)$. Each point represents a choice of parameters as in figure 2.
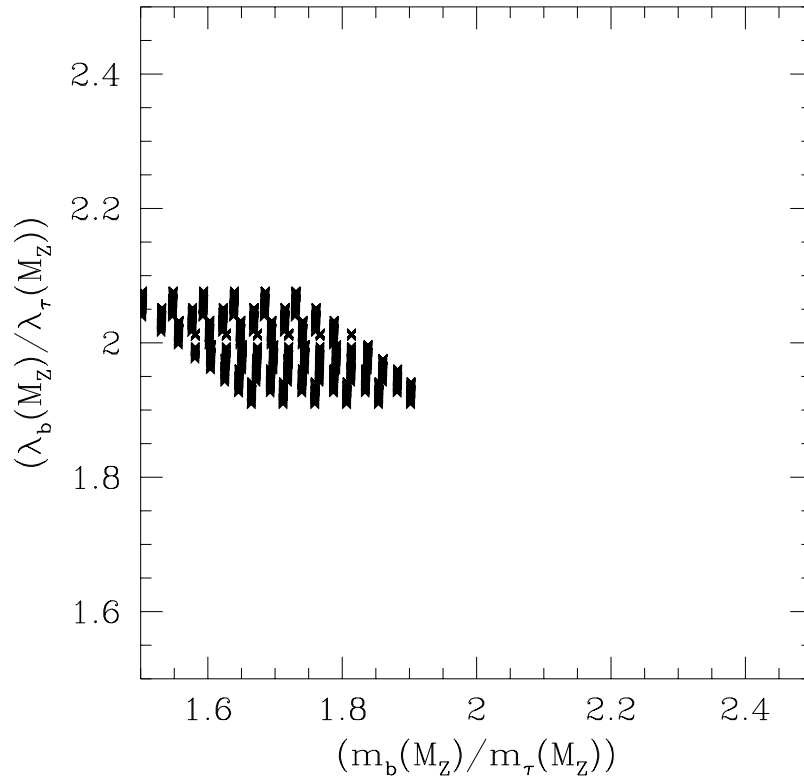
Figure 8. A scatter plot of the predicted ratio $(\lambda_b(M_Z)/\lambda_\tau(M_Z))$ versus the experimentally extrapolated mass ratio $(m_b(M_Z)/m_\tau(M_Z))$. All the superpartners are assumed to be degenerate at the Z mass scale. Each point represents a choice of parameters as in figure 2.
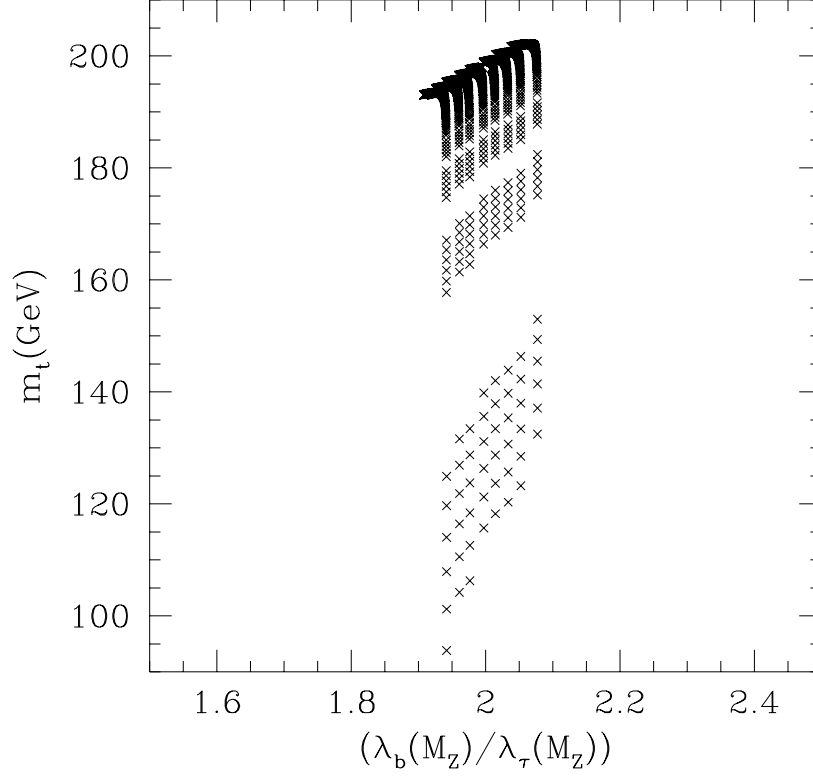
Figure 9. A scatter plot of the physical top quark mass, $m_t(m_t)$ versus the predicted ratio $(\lambda_b(M_Z)/\lambda_\tau(M_Z))$. Each point represents a choice of parameters as in figure 2. It is observed that there is no strong dependence of the predicted top quark mass on the ratio of the Yukawa couplings. Thus, the Yukawa ratio at the Z scale is mainly due to the QCD renormalization from the string scale to the weak scale while the predicted top quark mass mainly depend on the bottom quark Yukawa coupling, or alternatively on $\tan\beta$.
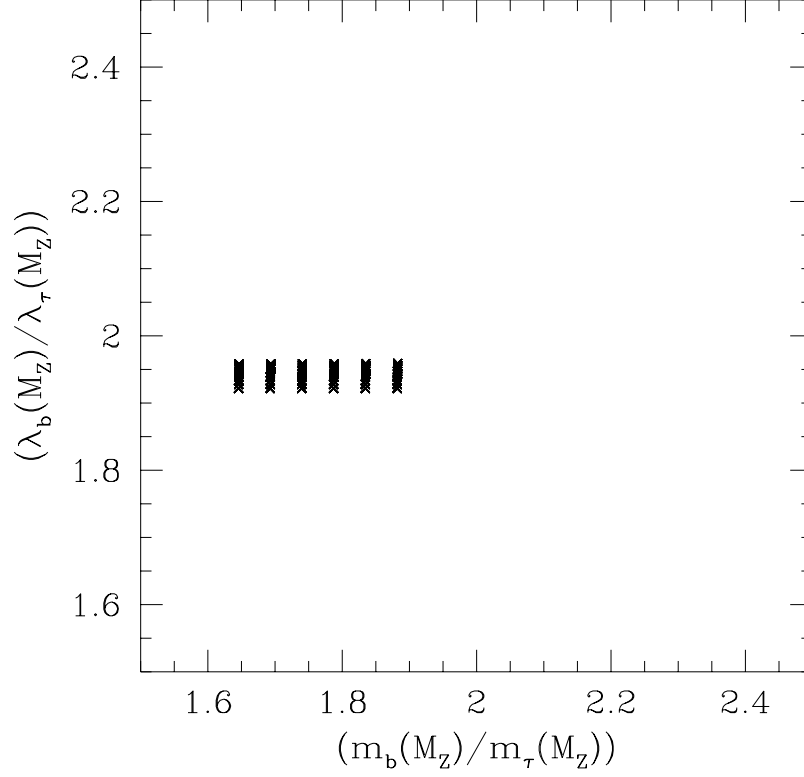
Figure 10. Scatter plot of the predicted ratio $(\lambda_b(M_Z)/\lambda_\tau(M_Z))$ versus the experimentally extrapolated ratio $(m_b(M_Z)/m_\tau(M_Z))$. Each point corresponds to a point in the parameter space. The range of the SUSY breaking parameters is given in table 3. The bottom quark mass is varied in the range $4.1 - 4.7$ GeV.
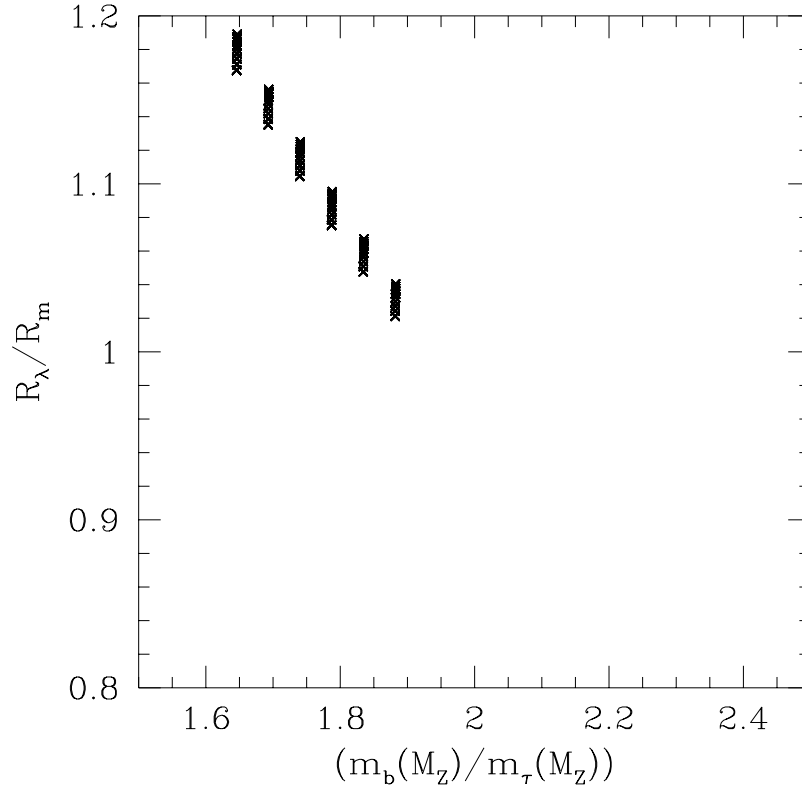
Figure 11. The ratio of $R_{\text{predicted}}/R_{\text{extrapolated}}$ versus $R_{\text{extrapolated}}$. Each point corresponds to a point in the parameter space of figure 10.